Coordination and Real Time Optimization of Signal Timing Plans for Urban Traffic Control

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Abstract – An effective method to improve traffic flow in urban areas is synchronizing the traffic signals at the coordinated intersections. This paper focuses on the problem of synchronization of subsequent intersections in a signalized urban area. Adopting an optimization model proposed in literature, the paper investigates the determination of the offset between two signals, i.e., the time displacements of green splits along a movement direction. On the basis of traffic observations, appropriate selection of offset in two coordinated intersections located in an urban area is performed under different congestion scenarios. Results show the efficiency of the proposed method to allow uninterrupted flow of traffic.

Keywords: signalized urban area, traffic control, offset determination, green phases optimization.

1 Introduction

The synchronization of traffic signals located along urban arterials in metropolitan areas is one of the most effective methods for improving traffic flow. The decision variables in a signal timing plan are cycle time, green splits, offsets and bandwidth [4]. Cycle time for a signal is defined as the duration of time from the center of the red phases to the center of the next red phase. Green splits for a signal in a given direction of movement is defined as the fraction of cycle time when the light is green in that direction. Several definitions have been employed for the offset between two signals in the literature. In particular, offset may be defined as the duration from the center of a green phase at one signal to the following nearest center (in time) of a green phase at the other signal. The offset determines the time displacements of green splits along a movement direction to allow uninterrupted flow of traffic.

In [3] the authors investigate the issue of urban traffic signal control for a signalized area including coordinated intersections. Starting from an optimization model proposed in the related literature [1], a modified model is proposed in order to take into account the changing traffic scenarios, the different types of vehicles pertaining to the chosen signalized area, as well as pedestrian movements in a unified framework. Although such a model is effective for real-time control purposes with reference to a generic urban area, it does not address the issue of coordinating local timing plans contributing to the overall network semaphoric cycle.

This paper focuses on the problem of synchronization of subsequent intersections in a signalized urban area in order to improve the effectiveness of the recalled model, i.e., to allow uninterrupted flow of traffic. In the related literature there is large evidence that inadequate offset determination greatly reduces the effectiveness of locally optimized signal timing plans. In other words, although a timing plan may be optimal for the whole area, the local semaphoric cycle of an intersection may disrupt the vehicles progression through the system because of local traffic demand [4]. In addition, an incorrect synchronization between successive intersections in the same direction may cause spillbacks phenomena: vehicles proceeding from one intersection to the downstream junction find only the concluding part of its green phase, and therefore line up in a queue which may produce oversaturation, blocking the upstream junction (the phenomenon is the so-called early return to green, [6]). Hence, synchronizing intersections located along the same urban network is an effective strategy to improve traffic flow in the signalized area [4]. This paper proposes a heuristic method to determine the offset value on the basis of the information on the actual traffic condition. Finally, a case study is analyzed and the obtained results show that the method is attractive for use in real applications.

The paper is organized as follows. Section II briefly reviews the model adopted in the sequel and section III shows the proposed strategy for coordinating signal timing plans. Moreover, section IV describes the case study and section V reports the results of the optimization performed under different traffic scenarios. Finally, section VI summarizes the main conclusions of the paper.
2 Modeling a signalized urban area

2.1 Purpose of the model

Signalized urban areas with automatic regulation of traffic lights are conceived to improve the performance of a traffic system, i.e., to alternate the right of way between traffic streams, minimize the average delay to vehicles and minimize occurrences of crash-producing conflicts. In particular, an optimal signal timing plan may be determined off-line on the basis of historic data, to be either periodically activated in fixed times of the day or dynamically selected on the basis of the actual traffic, or else it may be identified on-line.

The approach proposed in [1] and modified in [3] allows for modeling and controlling a generic urban area of signalized intersections both under congested and clear traffic. The proposed framework is based on a macroscopic model of the area, that limits the number of variables in the model and makes the strategy applicable in real time. The control task is to appropriately select the durations of green times in the pre-determined semaphoric cycle in order to minimize the number of vehicles in the system in the optimization horizon.

2.2 Review of the adopted model

A signalized urban area includes a number of coordinated intersections, i.e., junctions controlled by traffic lights sharing a common semaphoric cycle of length \( C \). The area includes a set \( L = \{L_i \mid i = 1, \ldots, I \} \) of \( I \) links. Each link models the space available between two subsequent traffic lights and may include one or several lanes. In particular, \( L_{in} \subset L \) and \( L_{out} \subset L \) respectively represent the sets of input links and output links, all with infinite capacity. In addition, set \( L \setminus (L_{in} \cup L_{out}) \) includes the finite capacity intermediate links, connecting intersections.

A generic link \( L_i \) with \( i = 1, \ldots, I \) is characterized by the following variables: \( n_i(k) \), \( N_i \), \( n_i(k) \) and \( y_i(k) \) with \( k = 1 \ldots K \), representing respectively the number of vehicles in link \( L_i \) at the beginning of the \( k \)-th semaphoric cycle in the pre-set optimization horizon (of length \( KC \)), the link capacity, the number of vehicles entering the link within the \( k \)-th cycle and the number of vehicles leaving it in the same time interval. Now, consider the traveling time \( \tau_i \) of \( L_i \) [3], and let \( l_i \) and \( v_i \) be respectively the link length and average vehicles speed (constant in the optimization horizon). Moreover, we call \( t(k) \) the duration of phase \( f \) in the \( k \)-th cycle.

A closed-form model of the signalized urban area may be determined on the basis of the above recalled variables. More precisely, for each link \( L_i \) (\( i = 1, \ldots, I \)) several vehicles balance equations may be derived [1, 3] with reference to the \( k \)-th cycle (\( k = 1 \ldots K \)), on the basis of the turning movement percentages \( \beta_{i,f} \) of vehicles traveling from \( L_i \) to \( L_o \), with \( L_i, L_o \in L \) [2]. In addition, in order to model the real traffic behavior in the area, the effective time \( t_{eff,i,j}(k) \) is defined [1, 3], representing the actual fraction of the \( f \)-th phase duration \( t(k) \) in the \( k \)-th cycle available for vehicles to leave \( L_i \) for \( L_j \). The model is completed by the definition of the following function:

\[
Q_{i,j}(t_{eff,i,j}(k)) = \phi_{i,j} \cdot t_{eff,i,j}(k)
\]

which represents the number of vehicles leaving from \( L_i \) to \( L_j \) during the \( f \)-th phase of the \( k \)-th cycle and takes into account the precedence constraints and the area topology [3]. We remark that (1) is a linear approximation of the actual variation of \( Q_{i,j} \) with \( t_{eff,i,j}(k) \), with parameter \( \phi_{i,j} \) representing the linear approximation slope and, ultimately, the current traffic scenario.

2.3 The optimization problem

An optimal signal timing plan includes appropriately selected green times \( t_{I'}(k) \) in the \( k \)-th cycle (\( k = 1, \ldots, K \)). The control objective is to minimize the number of vehicles in the system in the whole optimization horizon, i.e., to minimize the following objective function in the optimization horizon \( KC \):

\[
\min_{t_{I'}(k)} OF(K) = \min_{t_{I'}(k)} \sum_{i=1}^{I} \frac{1}{K} \left[ \sum_{k=1}^{K} n_i(k) \right]
\]

subject to the vehicles balance equations (see [1] and [3] for further details). In other words, solving the mathematical programming problem in equation (2) means finding an optimal duration of the green phases such that the average number of vehicles per cycle in the area is minimized. Now, we define a local objective function for each input and intermediate link, representing the mean number of vehicles per cycle in the link in the chosen optimization horizon:

\[
OF_i(K) = \frac{1}{K} \left[ \sum_{k=1}^{K} n_i(k) \right] , \quad i = 1, \ldots, I .
\]

The control task is to find the optimal green phases that minimize the summation of the local objective functions (3) in the area. An additional constraint is the following:

\[
(1-\delta) \cdot \hat{t}_{I'} \leq t_{I'}(k) \leq (1+\delta) \cdot \hat{t}_{I'}
\]

where \( f = 1, \ldots, F \), \( k = 1, \ldots, K \), \( \hat{t}_{I'} \) is the nominal duration of the green time for phase \( f \) and \( \delta \) represents the
maximum allowed percentage variation of such a nominal value. Finally, in order to fix the cycle length to the pre-set value C, the following constraint is included:

$$\sum_{f=1}^{F} t_f(k) = C, \quad k = 1, \ldots, K.$$  \hfill (5)

### 3 Coordination of signal timings

The model reviewed in the previous section is effective for real-time control purposes with reference to a generic urban area. However, it does not address the issue of coordinating local timing plans. This section addresses the problem of synchronizing subsequent intersections in a signalized urban area in order to allow uninterrupted flow of traffic.

The variables determining the time displacements of the green phases along a movement direction to allow uninterrupted flow of traffic are called offsets. In other words, the offset between two signalized intersections is defined as the duration from the center of a green phase at one signal to the center of the following nearest (in time) green phase at the other signal [4]. We remark that in its current form the proposed strategy to select offsets is designed to ensure progression for one direction.

Now, consider two successive intersections in one direction and suppose that link \( L_i \) in the area connects the two junctions (see figure 1). Let us call \( O_i \) the offset between the corresponding green phases. Most commonly in the literature the suggested offset is as follows:

$$O_i = \frac{l_i}{v_i},$$  \hfill (6)

where we recall that \( l_i \) and \( v_i \) are respectively the link length and average vehicles speed. In other words, the offset is selected as the ideal traveling time in the link. However, the previous expression is realistic when traffic is not congested. On the contrary, when vehicles line up in queues in the link lanes, the offset should take into account the time to drain the queue: in other words, \( u_i^{(t)}(k) \) in the link decreases due to congestion and (6) becomes impractical.

A more realistic selection of the offset may be derived as follows. Suppose that the connecting link \( L_i \) contains \( q_i \) vehicles lined up in a queue in one lane, as depicted in figure 1. We remark that multiple lanes do not impair the technique generality: if the link includes \( l_l \) lanes with turning movements split along different semaphoric phases, the sequel is still valid, since in each phase the link may be viewed as single-lane. However, if such a condition does not hold, then a different offset must be considered for each lane. Now, let us call \( d_a = 3 \) s and \( t_r = 1.1 \) s [5] the average vehicle acceleration time to reach the steady state speed and the driver reaction time, respectively. Moreover, let \( T_{ci} \) be the clearance time necessary for the queue of \( q_i \) vehicles to leave \( L_c \). Considering that the average vehicle length is 5 m, we get:

$$T_{ci}(q_i) = d_a + t_r \cdot q_i + \frac{5}{v_i} \cdot (q_i - 1).$$  \hfill (7)

Now, due to the presence of the queue, vehicles entering \( L_i \) find a free link section \( l_i - 5\tilde{q}_i \) (see figure 1) and a first approximation of their free traveling time before reaching the queue is as follows:

$$T_{fi}(q_i) = \frac{l_i - 5\tilde{q}_i}{v_i}.$$  \hfill (8)

However, while a vehicle enters \( L_i \) the queue moves forward, so that the free link section increases. In fact, during time \( \frac{l_i - 5\tilde{q}_i}{v_i} \) a number of \( \tilde{q}_i \) vehicles leaves the queue, which may be computed substituting \( T_{ci}(q_i) \) with \( \frac{l_i - 5\tilde{q}_i}{v_i} \) and \( q_i \) with \( \tilde{q}_i \) in (7):

$$\tilde{q}_i = \min \left\{ \text{round-down} \left( \frac{l_i - 5(q_i - 1) - d_a v_i}{v_i t_r + 5} \right), 0 \right\}$$  \hfill (9)

where round-down indicates the rounding down operation and the minimum accounts for no vehicle able to leave the link. Hence, the free link section is now \( l_i - 5(q_i - \tilde{q}_i) \) and the free traveling time is modified as follows:

$$T_{fi}(q_i) = d_a + \frac{l_i - 5(q_i - \tilde{q}_i)}{v_i}.$$  \hfill (10)

Reasoning as above, we could extend the iterative process to determine a better approximation of the free traveling time. However, considering that the length of an urban area link usually equals several tens of meters [5], it is reasonable to stop the process here. Now, taking into account that (10) underestimate the actual value of \( T_{fi} \), we improve the approximation including a heuristically determined 5% correction factor in (10):

$$T_{fi}(q_i) = 1.05 \left( d_a + \frac{l_i - 5(q_i - \tilde{q}_i)}{v_i} \right).$$  \hfill (11)

Hence, given a queue of \( q_i \) vehicles in link \( L_i \) connecting the two junctions, three different situations may occur when a vehicle enters the link.

1. \( T_{fi}(q_i) = T_{ci}(q_i) \). The free link traveling time
equals the queue clearance time. In other words, as soon as
the last vehicle in the queue leaves \( L_o \), the vehicle that has
entered the link from the upstream junction approaches the
downstream intersection without decelerating, i.e., at speed
\( v_c \). In such a case, if the green lights in the successive
junctions are simultaneously activated, i.e., if we select:

\[ O_i = 0, \quad (12) \]

then vehicles proceeding from the upstream intersection to
the downstream one do neither halt nor decelerate if the
green phase is still on.

2. \( T_{fi}(q_i)< T_{ci}(q_i) \). The free link traveling time is
lower than the queue clearance time. In such a case, if the
green lights in the successive junctions are simultaneously
activated, vehicles proceeding from the upstream
intersection to the downstream one have to stop and line
up at the end of the queue. Hence, in order to avoid
disruption of the traffic flow in the link, it is necessary to
bring the beginning of the green phase at the downstream
junction forward by the time interval \( T_{ci}(q_i) - T_{fi}(q_i) \).
Accordingly, if we assume that positive (negative) offset
values correspond to a postponement (brining forward) of
the green phase beginning, the recommended offset is:

\[ O_i = - \left( T_{ci}(q_i) - T_{fi}(q_i) \right), \quad (13) \]

3. \( T_{fi}(q_i)> T_{ci}(q_i) \). The free link traveling time is
greater than the queue clearance time. In such a case, in
order to avoid disruption of the traffic flow in the link and
let vehicles entering \( L_i \) find a green light, it is necessary to
postpone the beginning of the green phase at the
downstream junction and the recommended offset is:

\[ O_i = + \left( T_{fi}(q_i) - T_{ci}(q_i) \right), \quad (14) \]

Clearly, equations (12)-(13) and (14) may be
rewritten in compact form as follows:

\[ O_i = T_{fi}(q_i) - T_{ci}(q_i). \quad (15) \]

We remark that (15) represents a preliminary choice
for the offset between the timing plans of two coordinated
intersections, corresponding to a recommended choice for
the connecting links. The actual value of the offset may be
tune-tuning starting from (15) when further specifications
are considered, such as for instance the minimization of the
objective function \( OF(K) \) reported in section 2.

4 The case study

Figure 2 illustrates a signalized area located in the
urban district of Bari (Italy), including two intersections
and 13 links (see figure 2), of which two are intermediate
links \( (L_5, L_1) \) connecting the two junctions. The whole
area is usually congested in rush hours. In particular, links
\( L_5 \) and \( L_1 \) are usually filled up with extremely long queues,
so that the last vehicles in the queues generally cross the
corresponding intersection only after two semaphoric
cycles.

Clearly, traffic congestion phenomena differ with the
time of the day. Hence, in the sequel we consider different
timing plans for weekdays, and in particular: 07.30 - 10.00
am (Time Slot TS 1), 10.00 - 12.00 am and 05.00-07.00
pm (TS 2), 12.00 am - 02.00 pm (TS 3), 07.00 - 09.00 pm
(TS 4), and finally a plan for all the other times of the day
(TS 5). The two intersections in the area are controlled by
two different timing plans, that were merged into an
overall semaphoric cycle with \( C=105 \) s (not reported).

5 Coordination and optimization
results

The overall semaphoric cycle (not reported) is
obtained after synchronization of the intersections. More
precisely, an offset \( O_F=78 \) s, determined heuristically, is
present in the original semaphoric cycle between the
beginning of the green phase of \( L_5 \) and the beginning of
the green phase of \( L_1 \). The overall semaphoric cycle of the
area comprises 22 phases, including 13 amber phases and
3 all-red phases (all with pre-set duration, see [5]). Hence,
6 green phases are to be optimized, for each TS defined.

Table 1. Objective function values before
optimization for the case study \((K=15)\).

<table>
<thead>
<tr>
<th>Initial condition</th>
<th>Time slots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TS1</td>
</tr>
<tr>
<td>OF</td>
<td>98.3</td>
</tr>
<tr>
<td>( OF_{F(C=80)} )</td>
<td></td>
</tr>
<tr>
<td>( OF_{F(C=20)} )</td>
<td>22.8</td>
</tr>
<tr>
<td>( OF_{F(C=15)} )</td>
<td>8.4</td>
</tr>
<tr>
<td>( OF_{F(C=20)} )</td>
<td>2.7</td>
</tr>
<tr>
<td>( OF_{F(C=15)} )</td>
<td>0.1</td>
</tr>
<tr>
<td>( OF_{F(C=44)} )</td>
<td>53.7</td>
</tr>
<tr>
<td>( OF_{F(C=5)} )</td>
<td>3.0</td>
</tr>
<tr>
<td>( OF_{F(C=22)} )</td>
<td>0.5</td>
</tr>
<tr>
<td>( OF_{F(C=22)} )</td>
<td>7.1</td>
</tr>
</tbody>
</table>

The signalized area model and the mathematical
programming problem described in section 2 were
implemented in the Microsoft Excel software with the
Solver and Solver Table add-ons. Table 1 reports the
case study performance indices before optimization (i.e.,
with the synchronization of the intersections currently
implemented in the area). Each column in the table
 corresponds to a different time slot of the day. The second
line in the table shows the objective function values in the
different time slots computed for \( K=15 \) cycles. Moreover,
the subsequent lines report the local objective functions
values (3) determined with the original semaphoric cycle.
From an analysis of the table it is apparent that a large
disproportion holds between the local objective functions of $L_7$ and $L_5$. In particular, the connecting link $L_7$ contains on average only 0.6 vehicles per cycle, while its capacity equals $C=22$ vehicles, for a percentage occupation of about 3%. Link 5, on the contrary, is nearly full during most of the time slots; in addition, during TS4 (TS1) $OF_5$ even equals 64.5 (53.7), corresponding to a percentage occupation of about 147% (122%). In other words, during both TS4 and TS1 spillback occurs with reference to $L_5$.

An optimized semaphoric plan for the whole area was derived for each TS with the following approaches.

A1. The green phases were optimized solving (2) in the Excel framework with the synchronization currently implemented in the area ($O_2=78$ s).

A2. The offset value $O_2$ was computed for each TS according to (15), on the basis of knowledge of the average queue lengths in the connecting links, and the green phases were computed off-line accordingly.

A3. On the basis of heuristic conditions a fixed offset $O_2=86$ s was selected for each TS, and the green phases were optimized solving (2) in the Excel context.

A4. The offset value $O_2$ was computed for each TS according to (15), and the green phases were subsequently optimized solving (2) with the Excel Solver while keeping the validity of (15).

A5. The offset value $O_2$ was preliminarily computed for each TS according to (15), and the green phases were afterwards optimized solving (2), letting the Excel Solver modify the offset value chosen by (15).

Figure 3 reports the optimization results for TS4 under the above approaches. It is apparent that applying the optimization method (2) to the case study with an incorrect synchronization (approach A1) is not rewarding in terms of congestion. On the other hand, employing a correct synchronization with non-optimal green phases (A2) is not a successful strategy, either. A better approach is therefore to solve the optimization problem (2) after selecting a suitable offset either heuristically or on the basis of (15) (A3 and A4). Finally, figure 3 shows that the best strategy overall is to preliminarily synchronize the intersections on the basis of (15) and subsequently let the Excel Solver fine-tune the offset while minimizing (2).

In figure 4 the initial condition and the results obtained after synchronization and optimization (according to A5) are reported. Together with the overall objective function (2), the local objective functions for the input and intermediate links are reported. The improvement is apparent for almost each link. In addition, in figure 5 we compare the input and intermediate links utilization before and after application of the proposed method. The spillback phenomenon initially arising in $L_5$ and $L_5$ is removed and the capacity of $L_7$ is more utilized.

The model performance was evaluated by carrying out a robustness analysis in the presence of lane blockings due to accidents or maintenance works, i.e., for a reduction of some parameter $\phi_{i,j}$ in (1). As an example, consider $L_5$ during TS1, a time slot in which maintenance works are commonly carried out. The link includes two lanes and the allowed turning movements from $L_5$ are towards $L_{13}$ and $L_7$. The customary value of such parameter is $[3] \phi_{5,13} = \phi_{5,7} = 1.08 = \phi_{5}$. In order to evaluate the model performance, the area was optimized for different values of $\phi_5$, i.e., for scenarios in which one of the two lanes varies from completely blocked ($\phi_5=0.54$) to free ($\phi_5=1.08$). The corresponding objective function (2) is reported in figure 6 both for the non-optimized model as well as after synchronization and optimization. The indices decrease with an increase in $\phi_5$ (i.e., with an enhanced availability of one of the lanes), confirming the consistency of the model, with an optimized value of (2) always lower than the corresponding non-optimized one. In addition, the slope of the $\phi_5$ curve for the optimized model is considerably smaller than the corresponding slope of the non-optimized model, showing that the optimized area is less sensitive to lane blockings, i.e., to reductions in the links capacity.

6 Conclusions

This paper describes the development of a heuristic method to determine the offset value to improve coordination and synchronization of urban signal timing plans. A macroscopic model describes traffic in the signalized urban area and an optimization strategy proposed in related literature determines the signal timing plan. A case study considering a signalized urban area located in Bari (Italy) is analyzed. The results show the efficiency of the method to determine the offset value on the basis of the knowledge of the traffic condition. We remark that in its current form, the proposed strategy selects offsets to ensure progression for only one direction. Future research will address the determination of offsets to improve progression for both directions of a traffic network.

References


