Real Time Optimization of Traffic Signal Control: Application to Coordinated Intersections

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Abstract – This paper investigates the issue of urban traffic signal control using a real time optimization model for signalized areas proposed in the related literature. The adopted model is modified to take into account the traffic scenarios, the different types of vehicles in the area, as well as pedestrians. The technique is applied to a real case study, consisting of two coordinated intersections located in the urban area of Bari (Italy). On the basis of traffic observations, optimal selection of the phases in the semaphoric cycle is performed under different congestion scenarios. Results show the ability of the strategy to minimize the vehicle queue lengths in the area.

Keywords: Urban traffic, signalized area, traffic control, semaphoric cycle, green phases optimization.

1 Introduction

The increasing urban traffic congestion in major occidental cities and the reduction of urban space for the construction of new roads and roundabouts has lead traffic signal control to play a central role in nowadays urban traffic management. Despite the growing research on the topic, the problem of urban intersections congestion remains an open issue: even in cities where advanced traffic management systems are in operation, sudden changes in traffic flow caused by accidents or congested conditions due to rush hours are commonly managed by manually operated timing plans [3, 5].

In this paper the issue of urban traffic signal control is investigated for a signalized area including coordinated intersections. Starting from an optimization model proposed in the related literature [1], a complex topology area including two synchronized intersections is modeled and optimized. The proposed case study is a real junction, located in the urban area of Bari (Italy), with severe traffic congestion. A basic assumption of the technique is that traffic conditions are monitored by detectors. These feed information on the actual system state to the real time controller, that selects the optimal duration of green phases in the fixed semaphoric cycle in order to minimize the overall number of vehicles in the area. On the basis of real traffic observations, optimal selection of the phases in the semaphoric cycle is performed under different congestion scenarios. The model proposed in [1] is modified and applied to the case study in order to take into account the changing traffic scenarios, the different types of vehicles pertaining to the chosen signalized area, as well as pedestrian movements in a unified framework. In addition, the expression of a congested lane traveling time introduced in [1] is modified. Finally, amber and all-red phases are considered. The model performance is evaluated by carrying out a robustness analysis in the presence of lane blockings due to accidents or maintenance works.

This paper is organized as follows. Section II briefly reviews the model adopted in the sequel and section III shows the modifications to the technique. Moreover, section IV describes the case study and section V reports the results of the optimization performed under different traffic scenarios. Finally, section VI summarizes the main conclusions of the paper.

2 The adopted model

2.1 Purpose of the model

Signalized urban areas with automatic regulation of traffic lights are conceived to improve the performance of a traffic system, i.e., to alternate the right of way between traffic streams, minimize the average delay to all vehicles and pedestrians, as well as minimize the possibility of crash-producing conflicts. In particular, an optimal signal timing plan may be determined off-line on the basis of historic data, to be either periodically activated in fixed times of the day or dynamically selected on the basis of the actual traffic, or else it may be identified on-line.

The model proposed in [1] allows for modeling and controlling a generic urban area of signalized intersections both under congested and clear traffic. The approach is
based on a macroscopic model of the area, that limits the number of variables in the model and makes the strategy applicable in real time. The control task is to appropriately select the durations of green times in the semaphoric cycle (which is fixed and pre-determined) for the coordinated intersections pertaining to the area under study, on the basis of observations of traffic conditions. Hence, the overall control objective is to minimize the number of vehicles in the system in the whole optimization horizon, which is pre-set and equals K cycle lengths C.

2.2 Review of the model

Consider a generic signalized urban area comprising a number of coordinated intersections, i.e., junctions controlled by traffic lights pertaining to a common semaphoric cycle of length C, including a set \( L = \{ L_i \mid i = 1, \ldots, I \} \) of I links. Each link represents the space available between two subsequent traffic lights and may include one or several lanes. In particular, \( L_{in} \subseteq L \) and \( L_{out} \subseteq L \) respectively represent the sets of input links and output links, all with infinite capacity. In addition, the set \( L \setminus (L_{in} \cup L_{out}) \) includes the finite capacity intermediate links, connecting intersections in the area under study.

A generic link \( L_i \) with \( i = 1, \ldots, I \) is characterized by the following variables: \( n_i(k) \), \( N_i \), \( u_i(k) \) and \( y_i(k) \) with \( k = 1, \ldots, K \), representing respectively the number of vehicles in link \( L_i \) at the beginning of the \( k \)-th semaphoric cycle in the chosen optimization horizon (of length \( KC \)), the link capacity, the number of vehicles entering the link within the \( k \)-th cycle and the number of vehicles leaving it in the same time interval. The vehicles balance equation in link \( L_i \) (\( i = 1, \ldots, I \)) in the \( k \)-th cycle (\( k = 1, \ldots, K \)) is as follows:

\[
\frac{d n_i}{dt} = n_i(k + 1) = n_i(k) + u_i(k) - y_i(k) .
\]  

(1)

Clearly, traffic lights are associated only to input and intermediate links. In addition, let \( f = 1, \ldots, F \) be the generic phase associated to a traffic light in the semaphoric cycle, where \( F \) is the number of such phases. A phase is the time interval during which a given combination of traffic signals in the area is unchanged. Now, define for the generic links \( L_{in}, L_{out}, L_j \subseteq L \) \( S_{h_{in}}^f(k) \) and \( S_{j_{out}}^f(k) \) with \( f = 1, \ldots, F \) and \( k = 1, \ldots, K \), representing respectively the number of vehicles traveling from link \( L_{in} \) to \( L_{out} \) from \( L_i \) to \( L_j \) in the \( f \)-th phase of the \( k \)-th cycle. Then, defining \( u_{in}^f(k) \) and \( y_{out}^f(k) \) as the number of vehicles respectively entering and leaving from \( L_i \) in the \( f \)-th phase of the \( k \)-th cycle, it holds:

\[
\begin{align*}
 u_i(k) &= \sum_{f=1}^{F} u_{in}^f(k) \quad \text{and} \quad y_i(k) = \sum_{f=1}^{F} y_{out}^f(k) ,
\end{align*}
\]

(2)

\[
u_i^f(k) = \sum_{h \in L_{in}} S_{h_{in}}^f(k) \quad \text{and} \quad y_i^f(k) = \sum_{j \in L_{out}} S_{j_{out}}^f(k) .
\]

(3)

where \( L_{in} \) and \( L_{out} \) are respectively the set of incoming and outgoing links for \( L_i \). Moreover, let \( \beta_i, j \) be the percentage of vehicles traveling from \( L_i \) to \( L_j \). It holds:

\[
\sum_{j \in L_{out}} \beta_{i, j} = 1 \quad \forall (i, j) : j \in L_{out}^i .
\]

(4)

We assume that the turning movement fractions \( \beta_{i, j} \) are known and time variant. In particular, they are computed experimentally for the case study. It should be noted that such percentages may be estimated in real-time by known algorithms [2]. Now, consider the traveling time \( \tau_i = l_i / v_i \) of \( L_i \), where \( l_i \) and \( v_i \) are respectively the link length and average vehicles speed (assumed constant in the optimization horizon). In addition, let \( u_{in}^f(k) \) and \( u_{stop}^f(k) \) be the number of vehicles entering \( L_i \) in the \( f \)-th phase of the \( k \)-th cycle respectively able and unable to leave the link during the same phase. It holds:

\[
\begin{cases}
 u_{in}^f(k) = \begin{cases}
 u_{in}^f(k) \cdot \frac{\tau_i}{\tau_f} & \text{if} \quad \tau_i \leq \tau_f(k) ,
 0 & \text{if} \quad \tau_i > \tau_f(k) ,
\end{cases} \\
 u_{stop}^f(k) = u_{in}^f(k) - u_{in}^f(k) ,
\end{cases}
\]

(5)

\[
\begin{cases}
 n_{in}^f(k) = \begin{cases}
 n_i(k) + u_{in}^f(k) & \text{if} \quad f = 1 ,
 n_i(k) + u_{in}^f(k) + u_{stop}^f(k) - y_i(k) & \text{if} \quad f = 2, \ldots, F ,
\end{cases}
\end{cases}
\]

(7)

with \( i \in L \), \( f = 1, \ldots, F \), \( k = 1, \ldots, K \). In addition, \( \tau_f(k) \) is the duration of phase \( f \) in the \( k \)-th cycle. Under the assumption that the traveling time of the generic link is never greater than the length of two subsequent phases, define the variable \( n_{in}^f(k) \), representing the overall number of vehicles which can leave \( L_i \) during phase \( f \) of the \( k \)-th cycle, leaving aside the physical limitations imposed by the downstream links capacities (which are critical under congested traffic). It holds:

\[
\begin{cases}
 n_{in}^f(k) = \sum_{i = 1}^{F} n_{in}^f(k) + u_{in}^f(k) & \text{if} \quad f = 1 ,
 n_{in}^f(k) = n_{in}^{f-1}(k) + u_{in}^f(k) + u_{stop}^f(k) - y_i(k) & \text{if} \quad f = 2, \ldots, F ,
\end{cases}
\]

(6)
where \( X_{h,z} \) and \( V_{h,z} \) are the distance covered by the generic vehicle and its average speed while traveling from \( L_a \) to \( L_b \). In addition, \( R_{i,j} \) is the set of link pairs \((h,z)\) such that \( S_{h,z}^f(k) \) has right of way over \( S_{i,j}^f(k) \), with \( f \in \{1,...,F\} \). Hence, (8) models precedence in the area. Now, define the following function:

\[
Q_{i,j}(t_{eff_{i,j}}(k)) = \phi_{i,j} \cdot t_{eff_{i,j}}(k) 
\]

which represents the number of vehicles leaving from \( L_a \) to \( L_b \) during the \( f \)-th phase of the \( k \)-th cycle. The definition of such a function is crucial for an effective model of the actual traffic behavior, taking into account the precedence constraints and the area topology. In particular, (9) is a linear approximation of the actual variation of \( Q_{i,j} \) with \( t_{eff_{i,j}}(k) \), where parameter \( \phi_{i,j} \) represents the linear approximation slope and, ultimately, the current traffic scenario. The following equation completes the model in [1] by computing the number of vehicles \( S_{i,j}^f(k) \) going from link \( L_a \) to \( L_b \) in the \( f \)-th phase of the \( k \)-th cycle:

\[
S_{i,j}^f(k) = \min \left\{ \beta_{i,j} \cdot n_i^f(k), \beta_{i,j} \cdot p_s^f \cdot Q_{i,j}, N_j - n_i^f(k) + n_o^f(k) - \sum_{(h,z) \in R_{i,j}} S_{h,z}^f(k) + y_{i,j}^f(k) \right\} 
\]

with \( i \in L_a, \ j \in L_b, \ f = 1,...,F, \ k = 1,...,K \) and \( p_s^f \) representing the state of the traffic lights in \( L_a \) during phase \( f \) (1 for green and 0 for red light). We remark that in [1] amber lights are neglected. In the previous minimum expression the number of vehicles leaving \( L_a \) towards \( L_b \) comprises three factors, representing respectively: the number of vehicles directed to link \( j \) that are in link \( i \) at the beginning of the phase; the maximum number of vehicles transmitted to \( L_b \) taking into account the finite phase duration as well as the effective time due to precedence constraints; the maximum number of vehicles that \( L_b \) may accommodate in its finite capacity in the phase.

2.3 The optimization problem

An optimal signal timing plan includes appropriately selected green times \( t_i^f(k) \) in the \( k \)-th semaphoric cycle \((k = 1,...,K)\). The objective may be achieved by minimizing the number of vehicles in the system in the whole optimization horizon. Hence, the control task may be expressed by the mathematical programming problem [1]:

\[
\begin{align*}
\min & \quad \frac{1}{K} \sum_{k=1}^{K} \sum_{i \in L_b} n_i^f(k) \\
\text{subject to} & \quad (1), (2), (3), (4), (5), (6), (7), (8), (9) \text{ and } (10). \end{align*}
\]

An additional constraint is the following:

\[
(1-\delta) \cdot t_i^f(k) \leq t_i^f(k) \leq (1+\delta) \cdot t_i^f(k)
\]

where \( f = 1,...,F, \ k = 1,...,K \), \( t_i^f(k) \) is the nominal duration of the green time for phase \( f \) and \( \delta \) represents the maximum allowed percentage variation of such a nominal value. Clearly, (12) forces the optimization problem to reject solutions with extremely small or large green times, which may be hard to accept for regular drivers. Finally, in order to fix the cycle length to the pre-set value \( C \), the following constraint is included:

\[
\sum_{f=1}^{F} t_i^f(k) = C, \ k = 1,...,K.
\]

3 Modifications to the adopted model

In this section we modify the model [1], with emphasis on the vehicles crossing the area, the presence of pedestrians, the link traveling time, the number of vehicles entering an intermediate link and the traffic scenario.

3.1 Vehicles classification

In order to classify the different vehicles in the area, in the sequel the passenger car is adopted as the standard unit and other vehicles are assessed in terms of passenger car units (PCU) [4]. The classification of vehicles in PCUs is shown in table 1. Hence, by expressing vehicles in the area in terms of PCUs, we take into account the differences in dimensions, traveling times as well as accelerations of the vehicles.

<table>
<thead>
<tr>
<th>Type of vehicle</th>
<th>PCU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private cars, taxis, light private goods vehicles and trucks under 5 t</td>
<td>1.0</td>
</tr>
<tr>
<td>Motorcycles, scooters and mopeds</td>
<td>0.5</td>
</tr>
<tr>
<td>Buses, coaches and trucks over 5 t</td>
<td>3.0</td>
</tr>
<tr>
<td>Multiple axles rigid or articulated trucks</td>
<td>5.0</td>
</tr>
</tbody>
</table>

3.2 Amber lights and clearance times

In order to realistically model American and most European signalized urban areas, in the sequel we consider signal timing plans including green lights (signaling clear way), red lights (corresponding to a stop signal) and amber lights (corresponding to a caution signal after green and before red) [4]. In addition, we take into account the
so-called all-red phases or clearance times, i.e., short duration phases in which all traffic lights in one intersection are red, in order to let vehicles, previously allowed to occupy the crossing area and late due to congestion, clear the junction. Including such clearance phases in the semaphoric cycle is crucial when modeling congested areas with a complex topology; applying the present control strategy to such areas may produce considerable improvement, hence a realistic semaphoric cycle is essential. According to the Italian regulations for urban areas [4], we select fixed amber times of 4-5 s and fixed clearance times of 2 s.

3.3 Pedestrian crossings

In the sequel we take into account pedestrian crossings in order to realistically model traffic in a signalized urban area. Clearly, pedestrians can cross the intersection only when they find a green phase, and only in such case they may obstruct traffic. Hence, we consider pedestrians that have arrived at the intersection finding a red light together with strippers that arrive with a green light. Now, suppose that a vehicle and a pedestrian stream are simultaneously permitted in crossing directions. In such cases vehicles must give right of way to pedestrians, hence the green phase for the former is reduced. In the sequel we assume that the two opposite streams of pedestrians obstructing vehicular traffic are continuous and may thus be viewed as a unique flow with the total number of pedestrians moving in one direction.

If we consider two simultaneous crossing vehicle and pedestrian streams in link for the generic f-th green phase of the k-th semaphoric cycle, the phase is reduced as follows:

\[ t_{p,r} = t + \left( \frac{\sum S_{k}^{L} (k) n_{k,r} - p_{f}(k) l_{c}}{v_{p}} \right) \]

where \( p_{f}(k) \) represents the total number of pedestrians, \( l_{c} \) is the crosswalk length and \( v_{p}=1 \) m s\(^{-1} \) equals the average pedestrians speed [4]. Equation (14) states that the effective time for vehicles equals the green phase duration minus the summation of the crossing times of each pedestrian. However, such formula may be used only for infrequent and isolated pedestrian crossings, whereas in signalized urban areas pedestrians usually move grouped in platoons. In particular, pedestrians proceed with speed \( v_{p} \) in rows spaced 1 s in time, whereas the minimal length of each row equals 0.75 m [4]. So, if \( p_{i} \) is the number of pedestrians in one row, and \( n_{k} \) is the number of rows of pedestrians crossing the road in the k-th cycle, we get:

\[ p_{r} = \text{round} - \left( \frac{l_{c}}{0.75} \right) \quad n_{k}(k) = \text{round} - \left( \frac{p_{r}(k)}{p_{r}} \right) \]

where round-up indicates the rounding up operation.

Now, the first row of pedestrians that finds a green light crosses the street in a time:

\[ t_{p1} = \frac{l_{c}}{v_{p}} \]

The successive rows, distanced 1 s in time, require a time:

\[ t_{p,2} = \frac{l_{c}}{v_{p}} + 1; \quad t_{p,3} = \frac{l_{c}}{v_{p}} + 2; \quad \ldots \quad t_{p,r} = \frac{l_{c}}{v_{p}} + (n_{r} - 1) \]

where we omitted the dependency on k. Hence, we have:

\[ T_{p,r}(k) = \left( \frac{l_{c}}{v_{p}} + (n_{r}(k) - 1) \right) \cdot \text{sign}(n_{r}(k)) \]

where \( \text{sign}(n_{r}(k)) \) indicates the state of the pedestrians, i.e., 1 for presence of pedestrians and 0 for no crossings. Summing up, the effective time is as follows:

\[ t_{p,i,j}^{f} = \begin{cases} t(k) - \left( \frac{\sum S_{k}^{L} (k) n_{k,j} - p_{f}(k) l_{c}}{v_{p}} \right), & \text{if } t(k) > t_{p,r}(k) \\ 0, & \text{if } t(k) < t_{p,r}(k) \end{cases} \]

3.4 Traveling time

The traveling time for link \( L_{i} \) \( \tau_{i} = \frac{l_{i}}{v_{i}} \) [1] affects the computation of the number of vehicles entering the link and leaving it during the same phase. However, the previous expression is realistic when traffic is not congested. On the contrary, when vehicles line up in queues in the link lanes (see figure 1), the traveling time should take into account the time to drain the queue: in other words, \( u_{g}(k) \) in (5) decreases due to congestion.

Now, assume that vehicles entering \( L_{i} \) find a queue and suppose that the link comprises a single lane. Let us call \( d=3 \) s and \( t=1.1 \) s [4] the average vehicle acceleration time to reach the steady state speed and the driver reaction time, respectively. Moreover, let \( T_{i} \) be the time necessary for the queue to reach the steady state speed and \( q_{i} \) the number of vehicles in the queue. Hence, we get:

\[ T_{i} = d_{a} + t_{r} \cdot q_{i} \]

So, considering that the average vehicle length is 5 m [4], vehicles entering \( L_{i} \) find a free link section \( l_{i} - 5q_{i}l_{i} \) and a first approximation of the traveling time is \( \frac{l_{i} - 5q_{i}l_{i}}{v_{i}} \).

However, while vehicles enter \( L_{i} \), the queue moves forward, so that the free link section and the traveling time
increase. In fact, during time \( \frac{l_i - 5q_i}{v_i} \), a number of \( \tilde{q}_i \) vehicles leave the queue, which may be computed substituting \( T_i \) with \( \frac{l_i - 5q_i}{v_i} \) and \( q_i \) with \( \tilde{q}_i \) in (20):

\[
\tilde{q}_i = \min \left( \text{round-down} \left( \frac{l_i - 5q_i - d_a}{v_i} \right) \right), \quad (21)
\]

where round-down indicates the rounding down operation and the minimum accounts for no vehicle able to leave the link. Hence, the free link section is now \( l_i - 5(q_i - \tilde{q}_i) \) and the traveling time is modified as follows:

\[
\tau_i = \frac{l_i - 5(q_i - \tilde{q}_i)}{v_i} + T_{d_i}, \quad (22)
\]

where

\[
T_{d_i} = d_a + t_r \cdot (q_i - \tilde{q}_i) \quad (23)
\]

takes into account, according to (20), the time necessary to completely drain the remaining queue. In other words, (22) expresses the traveling time as the summation of the time necessary for the vehicle to reach the end of the moving queue and the time for this to be emptied.

Reasoning as above, we could continue the iterative process to determine a better approximation of the traveling time. However, considering that the length of an urban area link usually equals several tens of meters [4], it is reasonable to stop the process here. In fact, with the proposed approximation (22) for the traveling time, we neglect the vehicles that leave the queue in a time \( \tau_i = \frac{l_i - 5q_i}{v_i} \), that are \( \tilde{q}_i = \frac{l_i - 5q_i - d_a}{v_i} \) in number.

Experimental evidence showed that equation (22) determines a realistic approximation of the traveling time in the presence of queues with small dimension, i.e., when \( \frac{l_i}{v_i} \geq T_i \). On the contrary, for limited queues (or, equivalently, for extended links) the traveling time may be approximated by the expression proposed in [1]. Hence:

\[
\tau_i = \begin{cases} 
\frac{l_i - 5(q_i - \tilde{q}_i)}{v_i} + d_a + t_r \cdot (q_i - \tilde{q}_i) & \text{if } \frac{l_i}{v_i} < T_i \\
\frac{l_i}{v_i} & \text{if } \frac{l_i}{v_i} \geq T_i 
\end{cases} \quad (24)
\]

_Example 1._ Consider figure 1 with \( q=5 \), \( l=100 \) m and \( v_i=50 \) km h\(^{-1}\). The traveling time according to [1] is \( \tau_i = \frac{l_i}{v_i} = \frac{100 \cdot 3600}{30 \cdot 1000} = 7.2 \text{ s} \), whereas from (20) it holds \( T_i = d_a + t_r \cdot q_i = 3 + 1.1 \cdot 5 = 8.5 \cdot l_i / v_i \). By (21) and (24) we get \( \tilde{q}_i = \min \left( \text{round-down} \left( \frac{100 - 5 \cdot 5}{50 \cdot 0.28} \right) \right) = 2 \) and \( \tau_i = \frac{100 - 5(5 - 2)}{50 \cdot 0.28} + 3 + 1.1 \cdot (5 - 2) = 12.4 \text{ s} \), which is more realistic than the previous value.

Now, if the link has \( l_l \) lanes with turning movements split along different semaphoric phases, (24) is still valid, since in each phase the link may be viewed as single-lane. However, if such a condition does not hold, then (24) must be modified and a different effective time must be considered for each lane \( (l = 1, ..., l_l) \):

\[
\tilde{q}_{i,l} = \min \left( \text{round-down} \left( \frac{l_i - 5q_i - d_a}{v_i} \right) \right), \quad (25)
\]

\[
T_{i,l} = d_a + t_r \cdot q_{i,l}, \quad (26)
\]

\[
\tau_{i,l} = \begin{cases} 
\frac{l_i - 5(q_{i,l} - \tilde{q}_{i,l})}{v_i} + d_a + t_r \cdot (q_{i,l} - \tilde{q}_{i,l}) & \text{if } \frac{l_i}{v_i} < T_{i,l} \\
\frac{l_i}{v_i} & \text{if } \frac{l_i}{v_i} \geq T_{i,l}
\end{cases} \quad (27)
\]

### 3.5 Vehicles entering an intermediate link \( S^{f}_{L,f}(k) \)

Equation (10) computes the number of vehicles \( S^{f}_{L,f}(k) \) going from link \( L_i \) to \( L_j \) in the \( f \)-th phase of the \( k \)-th cycle [1]. Such a calculation includes a circular reference, since it requires knowledge of \( u^{f}_{j}(k) \) and \( u^{f}_{goj}(k) \), i.e., of the overall number of vehicles which can leave \( L_i \) during the phase and the number of vehicles
entering the link in that phase and leaving it during the same phase respectively, as well as of $u^f_i(k)$ and $y^f_j(k)$, i.e., of the number of vehicles respectively entering and leaving $L_j$ in the phase. All these variables involve knowledge of $S_{ij}^f(k)$ according to (3), (5) and (7). In order to avoid such a circular reference, we compute an initial approximation of $S_{ij}^f(k)$ by neglecting the third term in the minimum expression (10), i.e.:

$$S_{ij}^f(k) = \min\left\{ \beta_{ij}, n^f_i(k), p_{xi}^f, Q_{ij} \right\}. \quad (28)$$

Accordingly, we determine a first approximation of $n^f_i(k), u^f_{gij}(k), u^f_j(k)$ and $y^f_j(k)$ by substituting (28) in (7), (5) and (3) and finally applying (10). The method is iterative and may be stopped according to a stopping criterion (e.g., the number of required decimal numerals).

An alternative to the above method is to neglect in (10) the contribution of vehicles in the current phase and stop the computation to the $f$-th phase. The resulting estimate of $S_{ij}^f(k)$ is conservative and may be employed when spillbacks tend to obstruct the vehicles flow.

### 3.6 Traffic scenario (parameter $\phi_{i,j}$)

The number of vehicles leaving $L_i$ towards $L_j$ expressed by (10) comprises factor $\beta_{ij} \cdot p_{xi}^f \cdot Q_{ij}$, representing the maximum number of vehicles transmitted to $L_j$ taking into account the finite phase duration. Key parameters in the model are the terms $\phi_{i,j}$ in the effective time function $Q_{ij}$ (9). Such parameters represent the linear approximation slope in (9) and, ultimately, the current traffic scenario of a link. Hence, a correct value of $\phi_{i,j}$ is crucial for a realistic model of the signalized area.

Such parameters are generally time variant and may be computed experimentally, determining the number of vehicles that leave the link in the considered phase and dividing such number by the phase duration. An alternative is to approximate such parameters as follows.

Assume that $L_i$ comprises a single lane and that on average $n_{di}$ vehicles leave the link during its green phase $t'$. The first of these $n_{di}$ vehicles crosses the stop line in a time $t_{d1} = d_a + t_r$, while the following vehicles requires a time $t_{d2} = t_{d1} + t_r + \frac{5}{v_i}$, where the third term in the previous expression accounts for the average length of a vehicle of 5 m [4]. Hence, for the $n_{di}$-th vehicle it holds:

$$t' = t_{d1} + \left( n_{di} - 1 \right) t_r + \frac{S_{ij}^f(k) - \beta_{ij}}{v_i} = d_a + n_{di} t_r + \frac{5(n_{di} - 1)}{v_i}(29)$$

and trivial calculations lead to

$$n_{di} = \frac{v_i t' + 5 - d_a v_i}{v_i t_r + 5}. \quad (30)$$

We remark that in (30) the phase duration $t'$ includes the amber time, in which a few vehicles cross the intersection. Now, dividing (30) by the green phase we get:

$$\phi_{i,j} = \frac{n_{di}}{t'}. \quad (31)$$

Experimental evidence showed that under clear (saturated) traffic the above formula underestimates (overestimates) parameter $\phi_{i,j}$. Hence, a correction factor $\chi$ typically varying from 0.8 to 1.2 is applied:

$$\phi_{i,j} = \chi \frac{n_{di}}{t'}. \quad (32)$$

We remark that, although in [1] parameter $\phi_{i,j}$ is associated to a link $L_i$, if $L_i$ includes $l_l$ lanes then (32) should be modified and a single parameter $\phi_{i,l_1}$ with $l=1,...,l_l$ may be defined for each lane, unless the turning movements of the lanes are simultaneous.

**Example 2.** Consider figure 2, depicting a link with $l_{ll} = 3$ lanes. Assume that $C=100$ s is the cycle length and that lanes 2 and 3 have the same green phase with duration 40 s, while the turning movement associated to lane 1 takes place in a green phase with duration 20 s corresponding to the last 20 seconds of the green light for lanes 2 and 3. Since the turning movements of lanes 2 and 3 are simultaneous, the corresponding vehicle flows may be considered as a whole, and two parameters may be defined in the link, i.e., $\phi_{i,l_1}$ and $\phi_{i,l_2 \to 3}$. In addition, such parameters are multiplied in (10) by $\beta_{i,l_1}$ and $\beta_{l_2,j_2} + \beta_{i,l_3}$. In other words, each lane is viewed as an independent link with its own green phase, except for lanes with simultaneous movements, that may be grouped by considering an overall turning movements fraction.

### 4 The case study

Figure 3 illustrates the signalized area that was modeled and subsequently optimized to evaluate the technique. The proposed case study is a real junction, located in the urban area of Bari (Italy), with severe traffic congestion. In particular, the area consists of two synchronized intersections, regularly crossed by cars,
trucks, public transportation buses and mopeds. Moreover, a main vehicular direction cannot be identified in the area and all links are usually congested, with occasional spillback phenomena. More precisely, the signalized area (see figure 3) comprises 13 links, including 6 input links \((L_1, L_2, L_3, L_4, L_5, L_6)\), 5 output links \((L_8, L_9, L_{10}, L_{11}, L_{12})\) and 2 intermediate links \((L_7, L_{13})\). Although the whole area is usually congested in rush hours, the most troublesome links are \(L_9, L_5, L_6, L_7, L_{11}\). In particular, \(L_7\) and \(L_9\) are usually filled up with extremely long queues, so that the last vehicles in the queues generally cross the corresponding intersection after two semaphoric cycles. In addition, in \(L_7\) and \(L_{11}\), due to their limited extension (about 55 m), spillbacks tend to occur, so that vehicular traffic is nearly blocked. Clearly, such congestion phenomena differ with the time of the day. Hence, in the following optimization we consider different timing plans, and in particular: 07.30 - 10.00 am (Time Slot TS 1), 10.00 - 12.00 am and 05.00-07.00 pm (TS 2), 12.00 am - 02.00 pm (TS 3), 07.00 - 09.00 pm (TS 4), and finally a plan for all the other times of the day (TS 5). The two intersections in the area are controlled by two different semaphoric cycles, both of length \(C=105\) s, reported in figure 4. Note that amber lights and clearance times are included in the cycles.

5 Optimization results

In order to apply the method described in sections 2 and 3 to the case study, the semaphoric cycles of the intersections in the area (reported in figure 4) are merged. Hence, the overall semaphoric cycle (not reported) is obtained after synchronization of the intersections. More precisely, an offset of 8 s, determined heuristically, is introduced in the original semaphoric cycle between the beginning of the green phase of \(L_7\) and the beginning of the green phase of \(L_9\), in order to create a green wave between the two links [4]. In other words, with such an offset vehicles entering \(L_7\) for \(L_9\) find a moving queue with a quasi-uniform speed grouped in a platoon, with significant reduction of the area congestion. Choosing such an offset, i.e., synchronizing the two intersections, is crucial for the optimization of the area signal timing plan [1], since the method starts from a given semaphoric cycle, i.e., a given synchronization of the intersections. The overall semaphoric cycle of the area (not reported) comprises 22 phases, including 13 fixed-length amber phases and 3 all-red phases. Hence, 6 phases are optimized, for each time slot defined in weekdays. In particular, the signalized area model and the mathematical programming problem described in sections 2 and 3 are implemented in the Microsoft Excel software with the Solver and Solver Table add-ons.

Table 2 reports the results of the optimization for the case study: each row corresponds to a different time slot in the day. Moreover, columns 2 to 5 show the objective function value (11) determined with the original semaphoric cycle (i.e., with the synchronization of the intersections currently implemented in the area) and with the modified cycle (i.e., introducing an offset of 8 s), both for the non-optimized and optimized model. It is apparent from the table that for both cycles congestion is reduced with the proposed method. However, the improvement is more significant when the intersections synchronization is fine-tuned. The same conclusion may be drawn from figure 5, where the number of vehicles in \(L_5\) at the beginning of the \(k\)-th cycle \(n_k\) (during TS4) is reported as an example. For this reason, the inclusion of automatic synchronization in the optimization strategy is being investigated at present.

The model performance is evaluated by carrying out a robustness analysis in the presence of lane blockings due to accidents or maintenance works, i.e., for a reduction of some parameter \(\phi_{L_{i,j}}\). As an example, consider \(L_5\) during TS1, a time slot in which maintenance works are commonly carried out. The link includes two lanes and the allowed turning movements from \(L_5\) are towards \(L_{13}\) and \(L_7\). Application of (32) with \(\chi=1\) under the measured number of vehicles leaving the link shows that \(\phi_{L_{5,13}}=\phi_{L_{5,7}}=1.08=\phi_5\). In order to evaluate the model performance, the area is optimized for different values of \(\phi_5\), i.e., for scenarios in which one of the two lanes varies from completely blocked (\(\phi_5=0.54\)) to free (\(\phi_5=1.08\)). The corresponding objective function (11) is reported in figure 6 both for the non-optimized and optimized model. The indices decrease with an increase in \(\phi_5\) (i.e., with an enhanced availability of one of the lanes), confirming the consistency of the model, with an optimized value of (11) always lower than the corresponding non-optimized one.

6 Conclusions

In this paper the issue of urban traffic signal control is investigated for a signalized area including coordinated intersections. An optimization model proposed in the related literature [1] is modified and applied to a complex topology area including two synchronized intersections. Results show the ability of the real time control strategy to minimize the vehicle queue lengths in the intersection, even for traffic scenarios with lane blockings in the area.
The intent is to use the proposed approach in the future as a part of experimental real time control schemes. Moreover, the inclusion of automatic synchronization in the optimization strategy is presently being investigated.

References


