Design and Optimization of Integrated E-Supply Chain for Agile and Environmentally Conscious Manufacturing

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Abstract – An agile and environmentally conscious manufacturing paradigm refers to the ability to reconfigure a flexible system quickly, economically and environmentally responsibly. In modern manufacturing enterprises, e-supply chains integrate Internet and web-based electronic market and are promising systems to achieve agility. A key issue in the strategic logistic planning of integrated e-supply chains is the configuration of the partner network. This paper proposes a single- and multi-objective optimization model to configure the network of integrated e-supply chains. Considering an Internet-based distributed manufacturing system composed of different stages connected by material and information links, a procedure is presented to select the appropriate links. A set of performance indices are associated with the network links. Single-criterion and multi-criteria optimization models are presented under structural constraint definitions. The integer linear programming problem solution provides different network structures that allow us to improve the supply chain flexibility, agility and environmental performance in the design process. The proposed optimization strategy is applied to two case studies describing two networks for desktop computer production.

Index terms: Agile manufacturing, Internet-based manufacturing, Supply chain, Network design, Optimization model.

I. INTRODUCTION

Agile manufacturing has been introduced as a concept to satisfy the demand for low volume and high variety products. It requires embedding production flexibility into (re)configuration and control of manufacturing systems [28, 29]. By integrating computer systems, hardware and information flows, automated manufacturing systems provide agile manufactures with flexibility and reconfigurability. Flexibility is a manufacturing system’s ability to adjust to customers’ preferences and reconfigurability
is the ability to meet the changing demand by reconfiguring the system structure [10, 17, 29]. Customers’ expectation in terms of cost, quality, services, and environmental impact has put industries under pressure to become more agile and environmentally responsive and provide time and cost effective production realization deliveries under highly dynamic market and supply conditions. A common and accepted issue to achieve agility in a manufacturing system is manufacturing products in geographically different sites connected through communication networks [27]. In other words, an agile manufacturing system is composed of various resources that may belong to different companies and can be viewed as a complex Supply Chain (SC) network. More precisely, an SC network is defined as a collection of independent companies, possessing complementary skills and being integrated with streamlined material, information and financial flow [24]. Furthermore, such a system has to be dynamically reconfigurable according to product and market changes and has to provide high utilization of resources belonging to different companies.

The core of highly competitive and efficient SC networks is electronic information: Internet and web-based electronic market places can provide an inexpensive, secure and pervasive medium for information transfer between business units. In e-commerce, business, partners and customers connect together through Internet or other electronic communication systems to participate in commercial trading or to interact [3, 9, 11, 14]. Certainly, e-commerce poses new and hard issues in the company’s logistics systems, involving in some cases new distribution concepts and a new design of the SC. Hence, Integrated E-Supply Chain (IESC) is an emerging business strategy that integrates the supply chain design and the power of e-commerce in order to obtain more flexible and agile manufacturing processes [3, 8, 9, 11, 16, 19]. In the formation of an effective dynamic IESC network, the selection of partners in each tier of the SC for fulfillment of each order is extremely important [24]. In previous works [4, 5] the authors proposed a configuration methodology for IESC network design and partner selection. In order to face its complexity, the decision process is divided into three hierarchical levels: i) the candidate selection level, realized by different multi-criteria data analysis techniques; ii) the network design level, solved by multi-criteria optimization problems; and iii) the solution/validation level, which considers the impact of the higher levels on the operational layer. In [5] the authors focus particularly on the first level decision problems, i.e., the candidate selection module, and propose several criteria to select the partners of each stage on the basis of aggregate performance indices. This paper focuses on the second level problems, i.e., network design and optimization. In each IESC stage, one or more partners (producers, manufacturers, consumers etc.) have to be selected, making use of the information available for example on costs, transportation means, distances and pollution. Significant
literature deals with the problem of SC component selection and network design [7, 8, 13, 17, 22, 24, 25, 27] to achieve agility and flexibility. Indeed, among the problems encountered in strategic logistic planning, network configuration and design is considered the most significant issue [1, 13]. In particular, Wu et al. [27] formulate partner selection as an integer programming problem to choose one and only one candidate for each task of the production process. The authors minimize the sum of the costs for performing all the tasks and the transportation costs. However, the constraint to select only one candidate for each task is restrictive and affects the flexibility of the resulting network structure. Moreover, Jang et al. [13] propose an optimized supply network design module and a planning scheme of production and distribution activities that are modeled as three decomposed mathematical formulation. Each mathematical sub-model of the SC network is optimized by a Lagrangian heuristic and an integration methodology. However, the complete procedure appears complex and needs a genetic algorithm procedure to generate the final integrated production distribution plan. On the other hand, Viswanadham and Gaonkar [24] study and analyze a multi-tier SC by using an optimization technique which takes into account capacities and costs. In such a work, the authors determine the sub-optimal order quantities to be allocated to each of the manufacturers, suppliers and logistics service providers. A multi integer programming model is developed for a dynamic manufacturing network and its objective function maximizes the profit earned by the network subject to various capacity, production and logistics schedules and flow balancing constraints. However, information necessary to describe and characterize the SC can be so complex and large that the definition of constraints appears critical. To face such a complexity, Talluri and Baker [17] consider a three-phase approach to design an SC: phases I and II design the network and phase III addresses operational issues. However, the network design procedure does not consider the transportation connections among the stages in the network design and the routing of material is analyzed in the third phase only. Finally, Luo et al. [14] present a novel approach to describe and optimize an IESC network incorporating e-commerce and electronic linkages between actors with the associated environmental impact. The IESC network is composed by different consecutive stages connected by material and information links describing costs, distances, pollution and CO₂ emission. Moreover, the structure of the IESC is modeled by a digraph where nodes are stage partners and edges are links. Assigning different costs to the material links, Luo et al. [14] obtain the performance indices and the optimal flow material network by a fuzzy multi-objective optimization approach.

This paper proposes a configuration strategy for IESC network design, considering also the e-business relationships between operators, the network environmental impact and the de-manufacturing
to extract valuable raw materials and components from products [3, 8, 30, 31]. In particular, e-commerce is considered also between different non consecutive stages of the chain, i.e., between suppliers and consumers, manufacturers and retailers, etc. Indeed, the Internet encompasses a wide spectrum of potential commercial activities and influences the material transport and delivery among stages. The impact of e-commerce is here taken into account by the evaluation of particular performance measures assigned to the links connecting the stage partners [2, 18, 23]. Moreover, single criterion and multi-criteria objective problems are formulated to optimize the IESC network configuration considering at the same time material and information connections. Hence, an Integer Linear Programming (ILP) problem provides a set of possible alternative solutions. In addition, the IESC network is described by the digraph model proposed in [14] and the objective functions and constraints are characterized and defined on the basis of the digraph analysis. The presented methodology allows us to build the IESC network by selecting the edges of the digraph, for example to introduce e-commerce links and recycling links or to impose one or more manufacturers in the network. Consequently, comparing our methodology with the optimization technique proposed in [14], our formulation appears more flexible and suited to improve the IESC structure satisfying the different design requirements such as electronic links, material transporters, de-manufacturing, etc. The optimization model is applied to two case studies producing desktop computer systems. The first case study is solved in [14], hence the results obtained by the ILP are compared with the solutions inferred by the fuzzy-optimization approach. Moreover, in the second case study the set of obtained networks shows the improvement in flexibility and agility of the IESC structure.

This paper is organized as follows. In Section II the IESC structure and the associated digraph are described. Section III synthesizes the optimization model, that is described in detail for the two case studies in Sections IV and V. Finally, Section VI summarizes the conclusions.

II. THE INTEGRATED E-SUPPLY CHAIN NETWORK MODEL

A. The Integrated E-Supply Chain Description

An Integrated E-Supply Chain (IESC) network can be defined as a hyper-network of material flows overlaid with an e-business information network. The considered IESC contains different stages: raw material supply, intermediate supply, manufacturing, distribution, retail, customers, and de-manufacturing or re-cycling. For example, we consider inbound stages in which partners are raw material suppliers or plant stages. Moreover, the distribution stages have partners such as
manufacturers, product distributors and warehouses. Finally, outbound stages can be composed of retailers, customers, recyclers and de-manufacturers. After the de-manufacturing stage, recovered material, components or energy feedback to suitable Supply Chain (SC) stages are considered. In the IESC structure, Internet and e-business extend the benefits of conventional proprietary electronic data interchange systems to all the stages of the SC, leading to reduction in procurement cycles and costs, shortened product development times and reduced inventories. Moreover, e-business provides direct connections between final consumers and manufacturers.

We describe an IESC as a set of stages [14], denoted by $\mathbf{ST} = \{\mathbf{P}_1, \ldots, \mathbf{P}_k, \ldots, \mathbf{P}_{N_S}\}$, where $N_S$ is the number of stages. In addition, each stage $\mathbf{P}_k$ is described as a set of $s_k$ partners representing different actors of the IESC, i.e., $\mathbf{P}_k = \{n_{i_k}, n_{i_k+1}, n_{i_k+2}, \ldots, n_{i_k+s_k-1}\}$, where $i_k$ is the generic index such that $i_k = \sum_{h=1}^{k-1} s_h$ with $k=2,3,\ldots,N_S$ and $i_1=1$. We suppose that there are $N$ partners in the system. In addition, the (k-i)-th stage is called upstream stage of $\mathbf{P}_k$ and the (k+i)-th stage is called downstream stage of $\mathbf{P}_k$, with $i>0$. Moreover, the Bill Of Material (BOM) of stage $\mathbf{P}_k$ is a set of material and components required for processes in the k-th stage and produced by upstream stages.

Furthermore, the partners of different IESC stages can be connected by two types of links: material flow links ($m$-links) and information links ($e$-links). More precisely, an $m$-link represents the physical transportation link between two partners and multiple $m$-links are allowed between two partners to model different transportation modes or split delivery routes. In addition, a second type of link is the $e$-link, modeling e-business relationships between business entities for streamlining the material flow efficiently and effectively. We assume that two partners can be connected by $m$-links and/or by $e$-links and that an $e$-link may connect two partners of the SC also without the presence of an $m$-link. Hence, the proposed structure is able to extend the traditional SC into a more sustainable and integrated production system. Note that IESC partners belonging to the same stage are not connected by links, since in the considered model material and information flow though different stages. Formally, the $m$-links of stage $\mathbf{P}_k$ are denoted by the set $\mathbf{L}_{mk} = \{m_{ij}\}$, where $m_{ij}$ is an $m$-link starting from $n_i \in \mathbf{P}_k$ and ending in $n_j \in \mathbf{P}_h$, with $\mathbf{P}_k, \mathbf{P}_h \in \mathbf{ST}$ and $k \neq h$. The set $\mathbf{L}_m = \cup \mathbf{L}_{mk}$ denotes the overall material flow link set. Analogously, we denote the $e$-links set of stage $\mathbf{P}_k$ by $\mathbf{L}_{ek} = \{e_{ij}\}$, where $e_{ij}$ is an $e$-link starting
from \( n_i \in P_k \) and ending in \( n_j \in P_h \), with \( P_k, P_h \in ST \) and \( k \neq h \). The set \( L_e = \cup L_{e_k} \) denotes the \( e \)-links set and \( L = L_m \cup L_e \) is the complete set of links of the IESC. Figure 1 depicts a generic IESC network.

Moreover, we conclude the description of the IESC network by introducing the set of performance indices \( M = \{ M_1, M_2, \ldots, M_{N_M} \} \), where each element \( M_q \in M \) corresponds to a performance measure. Typical indices include cost, transportation and process time, product quality, energy consumption and environmental impact [2, 14]. A performance value is assigned to each link, considering \( m \)- and \( e \)-links: \( M_q(m_{ij}) \) (\( M_q(e_{ij}) \)) with \( q=1, \ldots, N_M \) denotes the value of the performance measure \( M_q \) associated with the link \( m_{ij} \in L_m \) (\( e_{ij} \in L_e \)). Particularly, an \( e \)-link speeds up the communication process and thus reduces the response time affecting performance measures such as costs and productivity. Hence, if two partners (for example \( n_i \in P_k \) and \( n_j \in P_h \)) are connected by both an \( e \)-link and an \( m \)-link (i.e., \( m_{ij} \) and \( e_{ij} \)), the performance measures are suitably updated and are associated with the \( m \)-link \( m_{ij} \) only. On the other hand, if just an \( e \)-link connects two IESC actors, then it models that fact that no material flow is possible between the two partners. In such a case, the performance measure that is assigned to the \( e \)-link is the cost of information, such as Internet portals, websites and electronic databases.

**B. Digraph Definition**

To exhibit the interactions among the stages of the IESC, we define a direct graph \( D=(N,E) \). The node set \( N \) represents the complete partner set of the network and each node \( n_i \in N \) for \( i=1, \ldots, N \) is associated with partner \( n_i \in P_k \) for \( k \in \{1, \ldots, N_S \} \) of the SC network. For the sake of simplicity, the same symbols indicate nodes and partners. Moreover, the edge set \( E \) is such that an arc \( y_{ij} \) directed from \( n_i \) to \( n_j \) is in \( E \) if there exists an \( m \)-link \( m_{ij} \in L_m \) and/or an \( e \)-link \( e_{ij} \in L_e \). We denote with \( E \) the number of edges in \( D \).
Example 1: Figure 2 shows an example of e-supply chain composed of $N_S=6$ stages: four suppliers, one manufacturer, two distributors, two retailers, one consumer and four recyclers, for a total of $N=14$ partners. The IESC exhibits the $m$- and $e$-links $m_{18}, e_{18}, m_{17}, e_{17}, m_{4,10}, e_{4,10}, m_{7,8}, e_{7,8}, m_{7,9}, e_{7,9}, m_{7,10}, e_{7,10}, m_{9,10}, e_{9,10}, m_{8,10}$, and $e_{8,10}$, while the remaining links are $m$-links. Moreover, Fig. 3 depicts the digraph describing the corresponding IESC network. The digraph $D=(N,E)$ has $N=14$ nodes and $E=23$ edges. Obviously, edges $y_{18}, y_{17}, y_{4,10}, y_{7,8}, y_{7,9}, y_{9,10}$ and $y_{8,10}$ are associated both with $m$- and $e$-links and the remaining edges of the digraphs are associated with $m$-links only. Moreover, each edge in $E$ is labeled by the corresponding variable $x_h$ with $h=1,...,E$, used in the optimization procedure and defined in the next section.

III. THE OPTIMIZATION MODEL

This section develops an optimization model to design the IESC network. The procedure starts from the knowledge of the digraph $D=(N,E)$, that takes into account all the possible actors belonging to the IESC stages and all the possible links that can connect the considered partners. Hence, an optimization technique has to select a subdigraph $D^*=(N^*,E^*)$ with $N^*\subset N$ and $E^*\subset E$, that corresponds to an IESC
responding to structural constraints and exhibiting optimal or suboptimal performance indices. To this aim, an Integer Linear Programming (ILP) problem is established considering a single-criterion or a multi-criteria optimization under a set of constraints obtained by the structure of the IESC. More precisely, the objective of the model is to minimize a single-criterion or multi-criteria cost function subject to the constraints that we characterize as BOM, path, mutual exclusion and structural constraints.

In order to state the optimization model and to use a clear mathematic notation, we introduce the decision variable vector \( x = [x_1, x_2, \ldots, x_E]^T \), where each element \( x_h \in \{0,1\} \) with \( h=1,\ldots,E \) is associated with an edge \( y_{ij} \in E \) (see the example in Fig. 3). More precisely, the value of \( x_h \) indicates the presence \((x_h=1)\) or the absence \((x_h=0)\) of the edge \( y_{ij} \in E \) in the solution digraph [12].

The objective function is obtained considering the performance index functions \( M_q \) that assign to each \( m\)-link \( m_{ij} \) and \( e\)-link \( e_{ij} \) the value \( M_q(m_{ij}) \) and \( M_q(e_{ij}) \), respectively, and to each \( m-e\)-link \( m_{ij}-e_{ij} \) the comprehensive value \( M_q(m_{ij}) \text{ or } M_q(e_{ij}) \). Consequently, value \( M_q(m_{ij}) \text{ or } M_q(e_{ij}) \) is associated with edge \( y_{ij} \in E \) and with the corresponding variable \( x_h \). Let us indicate with \( c^d = [c^d_1, c^d_2, \ldots, c^d_E]^T \) the vector of \( E \) entries where the \( h\)-th entry is \( c^d_h = M_q(m_{ij}) \) or \( c^d_h = M_q(e_{ij}) \) associated with the edge \( y_{ij} \in E \) and the decision variable \( x_h \) labeling \( y_{ij} \). The optimization problem is as follows:

\[
\begin{align*}
    z &= \min f(x) \quad (1) \\
    \text{subject to} \quad &Ax \geq B \quad (2) \\
    &x_h \in \{0,1\} \text{ for } h=1,\ldots,E \quad (3)
\end{align*}
\]

where \( A \) is the constraint matrix of dimension \( v \times E \) and \( B \) is a \( v \)-entry vector of integers, \( v \) representing the number of constraints. Minimizing the objective function \( f(x) \) means either to minimize only one performance index (Problem 1) or to minimize a subset of the chosen performance indices (Problem 2), i.e., \( f(x) \) represents either a single-criterion objective function or a multi-criteria objective function.

A. The Objective Function Definition

The single-criterion objective function of Problem 1. The single-criterion objective function is defined as follows:
Each solution $x^*$ of the ILP problem (1)-(4) for a particular vector $c^q$ corresponds to a possible IESC structure. More precisely, the optimal solution vector $x^*$ selects a sub-digraph $D^*=(N^*,E^*)$ of $D$. If the $h$-th entry of $x^*$ is $x^*_h=1$ and $x_h$ is associated to the edge $y_{ij}$ outgoing from $n_i$ and incoming to $n_j$, then the solution selects the edges $y_{ij} \in E^*$ and the nodes $n_i,n_j \in N^*$. In other words, the optimal IESC with respect to index $M_q$ is described by sub-digraph $D^*$ that exhibits the actors (nodes) and the links (edges) selected in the IESC design.

The multi-criteria objective function of Problem 2. The multi-criteria objective function is defined as follows:

$$f(x) = (c^q)^T x$$  \hspace{1cm} (4)$$

where $C = \begin{bmatrix} (c^{q1})^T \\ \vdots \\ (c^{qQ})^T \end{bmatrix}$ is a $Q \times E$ criteria matrix and $c^{q1}, \ldots, c^{qQ}$ are $E$-entry vectors associated with performance indices $M_{q1}, \ldots, M_{qQ}$, respectively.

The set of solutions of the multi-criteria ILP problem (1)-(3) and (5) provides the maximal Pareto face of the solutions set [6]. More precisely, we obtain a sub-set of solutions $X^* = \{x^*_i\}$ where each $x^*_i \in X^*$ is a Pareto optimal solution corresponding to a sub-digraph $D^*_i$ of $D$ and to an IESC structure.

B. Constraint Definition

BOM constraints. Each candidate supplier can provide a subset of materials (or components) to each producer. Analogously, the retailer actors provide a subset of products to each consumer. Generally, the BOM of each partner of the manufacturer stage or consumer stage is described as a list of specified materials needed by the stage. We distinguish between two cases: a) the manufacturer and the consumer stages include only one actor each (case A); b) the manufacturer and the consumer stages present more than one partner each (case B). The BOM constraints for both cases are defined as follows.
Case A) The actor of the considered stage $P_k$ (for example, manufacturer or consumer) has to obtain all the BOM components. For instance, let us suppose that the manufacturer requires all the components necessary to assemble the final product. In addition, suppose that each of these products can be equally shipped by means of three different edges respectively labeled with variables $x_1, x_2$ and $x_3$. This condition can be written as an inequality in 0-1 variables: $x_1 + x_2 + x_3 \geq 1$. Hence, if we suppose that there are $v_1$ BOM constraints, then the following inequality constraints are formulated:

$$A_1 x \geq \mathbf{1}$$

(6)

where $\mathbf{1}$ is a $v_1$-entry vector with all elements equal to 1 and $A_1$ is a $v_1 \times E$ constraint matrix defined by:

$$A_1 = \begin{bmatrix}
  a_1^1 \\
  a_1^2 \\
  \vdots \\
  a_1^v 
\end{bmatrix},$$

(7)

with $a_i^j = [a_i(i,1) \ a_i(i,2) \ \ldots \ a_i(i,E)]$ representing the $i$-th row of $A_1$ and $a_i(j) \in \{0,1\}$ for $j=1,\ldots,E$ and $i=1,\ldots,v_1$.

Fig. 4. Fictitious edges and nodes.

Case B) If there are $k$ actors in the considered stage $P_k$ (manufacturer or consumer) the designer has to select one or more partners in the stage. We model this situation by introducing a fictitious node $n_i$ and a fictitious edge labeled with variable $x_{f_i}$ with $i=k,\ldots,k+s_{k-1}$ associated with each node $n_i \in P_k$ (see Fig. 4). Hence, vector $x$ is modified introducing $k$ elements $x_{f_i}$ with $i=1,\ldots,k$: if entry $x_{f_i} = 1$ in
the solution vector $x^*$, then actor $n_i \in P_k$ is chosen. Summing up, the BOM constraints for each actor $n_i \in P_k$ are modified as follows:

$$a_i^T x \geq x_{f_i}, \text{ with } x_{f_i} \in \{0,1\}. \quad (8)$$

*Path constraints.* It is necessary to select in the digraph at least a path starting from a node of the producers and ending to the nodes of the consumers. To impose this condition, we associate with the digraph the $N \times E$ incidence matrix $I_M$ where each element is $I_M(i,j) \in \{-1,0,1\}$. More precisely:

- $I_M(i,j)=0$ if the arc labeled by $x_j$ does not belong to node $n_i$,
- $I_M(i,j)=-1$ if the arc labeled by $x_j$ starts from node $n_i$,
- $I_M(i,j)=1$ if the arc labeled by $x_j$ ends in node $n_i$.

Moreover, to define a constraint that imposes the presence of a path starting from node $n_h$ and ending at node $n_w$, we introduce the $N$-vector $b_{h,w}=[b_1, b_2, \ldots, b_N]^T$ with $b_h=-1$, $b_w=1$ and $b_p=0$ for $p \neq h,w$ and $p=1,\ldots,N$. The constraint is written as:

$$I_M x \geq b_{h,w}. \quad (9)$$

Hence, the constraint sub-matrix and the left side vector are:

$$A_2 = \begin{bmatrix} I_M \\ \vdots \\ I_M \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} b_{h1,w1} \\ \vdots \\ b_{hi,wi} \end{bmatrix}. \quad (10)$$

*Mutual exclusion constraints.* In some cases it is necessary to choose only one actor in a stage. In other words, the condition “at maximum one edge among those labeled with variables $x_1$, $x_2$ and $x_3$ can be in the solution digraph” is written with the following inequality in 0-1 variables: $x_1 + x_2 + x_3 \leq 1$. On the other hand, the condition “one and only one edge among those labeled with variables $x_1$, $x_2$ and $x_3$ has to be in the solution digraph” is expressed by $x_1 + x_2 + x_3 = 1$. Hence, $v_3$ mutual exclusion constraints can be expressed by the following equation:

$$A_3 x \leq 1 \quad (11)$$
where $\mathbf{1}$ is a $v_3$-entry vector with all elements equal to 1 and matrix $A_3$ is a constraint matrix defined by:

$$
A_3 = \begin{bmatrix}
a_3^1 \\
a_3^2 \\
\vdots \\
a_3^{v_3}
\end{bmatrix},
$$

(12)

with $\mathbf{a}_j^3=[d_j^3(i,1) \; d_j^3(i,2) \; \ldots \; d_j^3(i,E)]$ and $d_j^3(i,j)\in\{0,1\}$ for $j=1,\ldots,E$ and $i=1,\ldots,v_3$.

**Structural constraints.** Some particular structural constraints are related to a digraph. For example, the following condition can be imposed: “if edge corresponding to $x_1$ belongs to the solution digraph then edges corresponding to $x_2$ or $x_3$ belong to the solution digraph”. This condition is expressed by the constraint: $x_2+x_3\geq x_1$. A second type of structural condition is: “edge corresponding to $x_2$ belongs to the solution digraph if and only if (iff) edge labeled with variable $x_1$ belongs to the solution digraph”, i.e., $x_2=x_1$. Therefore, the $v_4$ structural constraints can be expressed by the following equations:

$$
A_4 x \geq \mathbf{0}
$$

(13)

$$
A_4 = \begin{bmatrix}
a_4^1 \\
a_4^2 \\
\vdots \\
a_4^{v_4}
\end{bmatrix},
$$

(14)

with $\mathbf{a}_j^4=[d_j^4(i,1) \; d_j^4(i,2) \; \ldots \; d_j^4(i,E)]$ and $d_j^4(i,j)\in\{0,1,-1\}$ for $j=1,\ldots,E$ and $i=1,\ldots,v_4$.

Finally, the constraint matrix and the specification vector are as follows:
The optimization models (1) and (4) or (5) subject to constraints (2) and (3) can be solved by applying standard algorithmic approaches for ILP problems [26]. However, we verified that the defined constraints give to the model a particular structure that plays an important role in the effective solution of the problem. Indeed, the constraints have a logical structure that falls into the case of the Shortest Path Problem [12, 26]. Consequently, the experimental results show that the Linear Programming (LP) relaxation of the problem gives an integer (0,1) solution. Hence, the proposed model is suitable to be straightforwardly solved applying standard algorithmic approaches for LP problems. Indeed, we verified that the integer solution is obtained by the standard two-phases simplex method [15]. On the other hand, the case studies considered in the following section model high level and aggregate real IESCs exhibiting typical optimization problem dimension. However, the rigorous proof of these results will be a subject of future research.

\[
A = \begin{bmatrix}
A_1 \\
A_2 \\
-A_3 \\
A_4
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
1 \\
B_2 \\
-1 \\
0
\end{bmatrix}.
\] (15)

\[\text{C. Application Methodology}\]

The proposed IESC optimization model is an instrument to design the supply chain so that some required issues are satisfied. More precisely, the steps that the decision makers have to accomplish are the following.

Step 1. Identify the supply chain stages and all the possible partners that can compose them.

Step 2. Determine all the possible links that can connect the partner stages, i.e., single out all the possible material and information connections among the IESC partners.

Step 3. Identify the performance indices that can characterize each material connection (e.g., costs, environment impact, type of transporters, etc.) and each electronic link (e.g., costs). If two partners are connected with an \(m\)-link and an \(e\)-link, then establish how the \(e\)-business information can influence the material connection in terms of the defined performance indices. This step is very important and it is crucial for the good results of the procedure. The availability of qualified expert personnel and accurate historical data base about IESC components can help in accomplishing this task.
Step 4. Define the optimization model. More precisely, select the performance indices to optimize and impose the suitable constraints. By this step the designers decide IESC structural characteristics (i.e., the presence of recycling, one or more manufacturers, e-commerce connections, etc.).

Step 5. Perform the optimization algorithm so that only one solution (in the case of Problem 1) or a set of solutions (in the case of Problem 2) is obtained.

IV. CASE STUDY 1

To illustrate the network design optimization procedure, we consider a case study inspired by an example proposed in [14]. The target product is a typical desktop computer system consisting of the computer, hard disk driver, monitor, keyboard and mouse. The IESC network corresponds to the network described in Example 1 and depicted in Fig. 2, while its digraph is shown in Fig. 3. The data for the case study are reported in Table I [14], that shows the values of each performance index $M_q$ with $q=1,\ldots,4$ associated with the links of the considered IESC. More precisely, the adopted performance indices are total costs ($M_1$), energy ($M_2$), CO$_2$ emission ($M_3$) and cycle time ($M_4$). We indicate generically by cycle time associated with an $m$-link or $m$-$e$-link the related time required by the transportation and/or the production process. The considered performance index values are reported in Table I and depend on the type of link ($m$- and $e$-link or $m$-link), the distance between the connected SC partners, the transportation mode (truck, car, airplane etc.) and the type of material to be transported. In particular, the costs and energy performance indices reported in the last two rows of Table I, respectively associated with links $m_{11,5}$ and $m_{14,3}$ are negative. In fact, in the recycler stage $P_6$ partner $n_{11}$ is a de-manufacturer, with an output link $m_{11,5}$ connecting to manufacturer $n_5$, and partner $n_{14}$ is a material recoverer, with an output link $m_{14,3}$ connecting to supplier $n_3$ (see Fig. 2). Hence, the total costs and energy associated with links $m_{11,5}$ and $m_{14,3}$ are negative, i.e., they correspond to recycling material and parts.

Various computational experiments are performed to minimize costs, energy consumption, CO$_2$ emission and Total Lead Time (TLT), alternatively. In particular, the TLT is defined as the total time elapsed from the instant at which the raw material begins its travel until the instant the finished product is delivered to consumers. Furthermore, a multi-objective function for Problem 2 is chosen. The solutions are obtained implementing the well-known two-phase simplex method in the Matlab framework [20, 21].
### TABLE I.

**DATA SHEET FOR THE NETWORK LINKS IN CASE STUDY 1.**

<table>
<thead>
<tr>
<th>Links</th>
<th>Edges</th>
<th>Variables</th>
<th>Costs (M₁) in US$</th>
<th>Energy (M₂) in MJ</th>
<th>CO2 emission (M₃) in KgCE</th>
<th>Cycle time (M₄) in hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>m₁₈↔₄₈</td>
<td>Y₁₈</td>
<td>x₁</td>
<td>41.80</td>
<td>359.00</td>
<td>0.87</td>
<td>19.30</td>
</tr>
<tr>
<td>m₁₅,₇↔₁₇</td>
<td>Y₁₇</td>
<td>x₂</td>
<td>46.70</td>
<td>332.00</td>
<td>0.74</td>
<td>16.80</td>
</tr>
<tr>
<td>m₁₃</td>
<td>Y₁₃</td>
<td>x₃</td>
<td>319.00</td>
<td>1479.00</td>
<td>2.21</td>
<td>12.50</td>
</tr>
<tr>
<td>m₁₂</td>
<td>Y₁₂</td>
<td>x₄</td>
<td>308.00</td>
<td>1776.00</td>
<td>2.19</td>
<td>12.80</td>
</tr>
<tr>
<td>m₁₁</td>
<td>Y₁₁</td>
<td>x₅</td>
<td>238.00</td>
<td>1540.00</td>
<td>3.10</td>
<td>16.20</td>
</tr>
<tr>
<td>m₁₀</td>
<td>Y₁₀</td>
<td>x₆</td>
<td>246.00</td>
<td>1409.00</td>
<td>1.47</td>
<td>10.20</td>
</tr>
<tr>
<td>m₊₉,₄₀,₄₁,₁₀</td>
<td>Y₄₁₀</td>
<td>x₇</td>
<td>53.90</td>
<td>369.00</td>
<td>30.20</td>
<td>5.30</td>
</tr>
<tr>
<td>m₉₆</td>
<td>Y₆₈</td>
<td>x₈</td>
<td>448.00</td>
<td>3618.00</td>
<td>8.74</td>
<td>19.20</td>
</tr>
<tr>
<td>m₈₅</td>
<td>Y₈₅</td>
<td>x₉</td>
<td>379.00</td>
<td>3542.00</td>
<td>296.00</td>
<td>4.20</td>
</tr>
<tr>
<td>m₈₄</td>
<td>Y₈₄</td>
<td>x₁₀</td>
<td>358.00</td>
<td>2885.00</td>
<td>6.26</td>
<td>16.20</td>
</tr>
<tr>
<td>m₈₃</td>
<td>Y₈₃</td>
<td>x₁₁</td>
<td>358.00</td>
<td>3259.00</td>
<td>223.00</td>
<td>3.90</td>
</tr>
<tr>
<td>m₈₂</td>
<td>Y₈₂</td>
<td>x₁₂</td>
<td>20.89</td>
<td>13.40</td>
<td>0.87</td>
<td>121.70</td>
</tr>
<tr>
<td>m₇₈,₇₉,e₉₈</td>
<td>Y₇₈</td>
<td>x₁₃</td>
<td>25.20</td>
<td>16.40</td>
<td>1.10</td>
<td>123.00</td>
</tr>
<tr>
<td>m₇₇,₇₂,₇₁₀</td>
<td>Y₇₁₀</td>
<td>x₁₄</td>
<td>22.90</td>
<td>35.10</td>
<td>2.58</td>
<td>65.80</td>
</tr>
<tr>
<td>m₇₆,₇₉,e₇₈</td>
<td>Y₇₉</td>
<td>x₁₅</td>
<td>20.70</td>
<td>9.18</td>
<td>0.59</td>
<td>61.30</td>
</tr>
<tr>
<td>m₆₈,₆₉,₆₁₀</td>
<td>Y₆₁₀</td>
<td>x₁₆</td>
<td>64.00</td>
<td>90.40</td>
<td>0.56</td>
<td>120.30</td>
</tr>
<tr>
<td>m₅₆,₅₇,₅₁₀</td>
<td>Y₅₁₀</td>
<td>x₁₇</td>
<td>58.10</td>
<td>4.68</td>
<td>0.13</td>
<td>100.00</td>
</tr>
<tr>
<td>m₄₈,₄₉,₄₁₀</td>
<td>Y₄₁₀</td>
<td>x₁₈</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
<td>0.80</td>
</tr>
<tr>
<td>m₃₁,₈,₃₁₂</td>
<td>Y₃₁₂</td>
<td>x₁₉</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
<td>0.80</td>
</tr>
<tr>
<td>m₂₃,₃₁₃</td>
<td>Y₃₁₃</td>
<td>x₂₀</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
<td>0.80</td>
</tr>
<tr>
<td>m₁₄,₄₁,₄₂</td>
<td>Y₄₂</td>
<td>x₂₁</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
<td>0.80</td>
</tr>
<tr>
<td>m₁₃,₅</td>
<td>Y₁₃₅</td>
<td>x₂₂</td>
<td>-18.00</td>
<td>-11.00</td>
<td>0.74</td>
<td>4.80</td>
</tr>
<tr>
<td>m₁₂,₄</td>
<td>Y₁₂₄</td>
<td>x₂₃</td>
<td>-28.00</td>
<td>-6.60</td>
<td>1.10</td>
<td>6.50</td>
</tr>
</tbody>
</table>

#### A. Constraint Definition

**BOM constraints.** The component supplier constraints are obtained taking into account that the BOM of the second stage in Fig. 2, representing the manufacturer, is the following: computer (C), hard-disk-driver (H), monitor (M), keyboard/mouse (K). We assume that C is produced by n₁ and n₂, H is produced by n₁, n₂ and n₃, M is produced by n₂, n₃ and n₄, and K is produced by n₃ and n₄ [14]. Hence, with reference to Fig. 3 the constraints imposed on the variables labeling the edges are as follows:

\[ x₃ + x₄ ≥ 1 \]
\[ x₃ + x₄ + x₅ ≥ 1 \]
\[ x₄ + x₅ + x₆ ≥ 1 \]
\[ x₅ + x₆ ≥ 1 \]  \hspace{1cm} (16)

**Path constraints.** Case study 1 includes only one manufacturer and only one consumer (node n₅ of stage P₂ and n₁₀ of P₅, respectively, in Fig. 2). Hence, a path between nodes n₅ and n₁₀ is needed.
Consequently, we build the N×E incidence matrix $I_M$ associated with digraph $D$. Moreover, we define the 23-vector $b_{5,10}=\{b_1, b_2, \ldots, b_{23}\}$ with $b_5=-1$, $b_{10}=1$ and $b_p=0$ for $p \neq 5, 10$ and $p=1,\ldots,23$. The constraint that imposes the presence of a path starting from node $n_5$ and ending in node $n_{10}$ is written as follows:

$$I_M x \geq b_{5,10}$$  \hspace{1cm} (17)

**Mutual exclusion constraints.** It is assumed that the design conditions of the SC network include the hypothesis that one and only one partner is to be included in the recycler stage (stage $P_6$ in Fig. 2). Furthermore, only one type of commerce is present between the second and third stages and one and only one $m$-$e$-link is present among the first stage and the others. Hence, with reference to Fig. 3, the mutual exclusion constraints are the following:

$$
\begin{align*}
&x_{18} + x_{19} + x_{20} + x_{21} \leq 1 \\
&x_{13} + x_{14} + x_{15} \leq 1 \\
&x_1 + x_2 + x_7 = 1
\end{align*}
$$  \hspace{1cm} (18)

**Structural constraints.** The constraints derived from the digraph structure in Fig. 3 are as follows:

$$
\begin{align*}
x_{22} - x_{18} &= 0 \\
x_{23} - x_{21} &= 0 \\
x_5 - x_{23} &\geq 0 \\
x_{16} - x_1 &\geq 0 \\
x_{13} + x_{14} + x_{15} - x_2 &\geq 0 \\
x_{16} - x_{13} &\geq 0 \\
x_{17} - x_{15} &\geq 0
\end{align*}
$$  \hspace{1cm} (19)

For example, the first constraint of (19) means that the edge corresponding to $x_{22}$ is selected iff the edge labeled by $x_{18}$ is selected. In addition, the third constraint of (19) means that if the edge labeled by $x_{23}$ is selected then the edge corresponding to $x_5$ is selected.
B. Solution of Problem 1

Problem 1 is solved with respect to four objectives alternatively: costs, energy, CO₂ emission and TLT. The corresponding objective functions are denoted by $f_1$, $f_2$, $f_3$ and $f_4$, respectively. The obtained sub-digraphs are presented in Fig. 5, 6, 7 and 8 respectively, and the corresponding objective functions are given in Table II.

![Fig. 5. Solution digraph of min($f_1$).](image1)

![Fig. 6. Solution digraph of min($f_2$).](image2)

![Fig. 7. Solution digraph of min($f_3$).](image3)

![Fig. 8. Solution digraph of min($f_4$).](image4)

Both the solution digraphs respectively minimizing costs and energy include a recycler (i.e., $n_{14}$ in Fig. 5 and $n_{11}$ in Fig. 6), which is missing in the network that minimizes CO₂ emission and TLT depicted in Fig. 7 and 8, respectively. This result is obvious, since including the recycling stage in the
IESC network results in a reduction of costs and energy (notice the negative entries in the last two rows of Table I) but does not decrease CO$_2$ emission and TLT.

**TABLE II.**

THE VALUES OF OBJECTIVE FUNCTIONS $f_1, f_2, f_3$ AND $f_4$ FOR PROBLEM 1 OF CASE STUDY 1.

<table>
<thead>
<tr>
<th></th>
<th>$f_1$ (US$)</th>
<th>$f_2$ (MJ)</th>
<th>$f_3$ (KgCE)</th>
<th>$f_4$ (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min $f_1$</td>
<td>946.02</td>
<td>6566.30</td>
<td>16.34</td>
<td>98.20</td>
</tr>
<tr>
<td>Min $f_2$</td>
<td>1030.92</td>
<td>6112.66</td>
<td>12.51</td>
<td>190.00</td>
</tr>
<tr>
<td>Min $f_3$</td>
<td>1037.50</td>
<td>6415.86</td>
<td><strong>11.38</strong></td>
<td>190.30</td>
</tr>
<tr>
<td>Min $f_4$</td>
<td>997.90</td>
<td>6799.00</td>
<td>329.88</td>
<td><strong>16.70</strong></td>
</tr>
</tbody>
</table>

Comparing our results with the solutions reported in [14], we remark the following two aspects. First, while the result of the optimization problems min($f_2$) and min($f_3$) provide the same results as [14], the minimization of the objective functions $f_1$ and $f_3$ does not provide the same digraphs obtained in [14]. Indeed, the fuzzy optimization can lead to suboptimal solutions: the optimal value of costs and CO$_2$ emission obtained with ILP is $f_1$=946.02 US$ and $f_3$=11.38 KgCE respectively, while the fuzzy optimization performed in [14] determines two solutions with $f_1$=951.00 US$ and $f_3$=14.10 KgCE, respectively. Consequently, the ILP approach with single criterion objective function guarantees optimal solutions.

Second, in [14] the authors use the same structure of BOM constraints for all the considered performance indices but such a structure is not suited to the TLT performance measure. Indeed, the cycle time associated with BOM constraints is not the sum of the corresponding edge performance indices but the maximum among the performance indices. For example, if we choose the edges $y_{25}$ and $y_{35}$ corresponding to the variables $x_4$ and $x_5$ as BOM for $P_2$, the corresponding cycle time can not be computed as $M_4(m_{25})+M_4(m_{35})$, but as the maximum between $M_4(m_{25})$ and $M_4(m_{35})$: in such a case the constraint becomes non-linear. Hence, to obtain a more rigorous model but with linear constraints, we modify the constraints (16) to transform the non-linear BOM constraints for the TLT in suited linear constraints [15]. However, since the cycle times assigned to links $m_{ij}$$\in L_m$ do not differ much, in this particular case the assumption used in [14] is admissible and we obtain the same solution digraph of problem min($f_2$) (see Fig. 8).

Finally, the following example shows that the presented optimization method can improve the reconfigurability of the network. In particular, let us consider a traditional SC which has a network composed of $m$-links only (see Fig. 9). Moreover, the designer has to add the e-links in order to introduce e-commerce and e-business fixtures in the network structure optimizing the costs.
Consequently, Problem 1 is solved by selecting costs as performance index subject to the constraints (16)-(19) and the following mutual exclusion constraints that impose the initial structure of the SC:

\[ x_4 = x_5 = x_9 = x_{19} = 1 \]  

(20)

The resulting network is depicted in Fig. 10 and exhibits a cost of 979.32 US$.

Fig. 9. The digraph structure of a traditional SC composed of \( m \)-links.

Fig. 10. Solution digraph of \( \min(f_i) \) imposing a fixed structure of \( m-e \)-links.

C. Solution of Problem 2

The multi-objective optimization problem is solved considering the following performance indices: costs, energy and CO\(_2\) emission (\( f_3 \)). According to the previous remarks, we do not consider the cycle time in the multi-objective optimization, but we compute the TLT values of the problem solutions. Table III reports the ILP solutions and the corresponding values of the performance indices. The results show the efficiency of the proposed method, that is able to provide a set of optimal solutions. For example, solution \( x_A \), obtained minimizing objective function \( f_3 \), is equal to the solution obtained minimizing \( f_1 \) (compare the first row of Table III with Fig. 5). Table III shows that solution \( x_A \) exhibits a large value of CO\(_2\) emission. On the other hand, minimizing objective function \( f_3 \) provides solution \( x_D \) in Table III, featuring satisfactory values of costs, energy, CO\(_2\) emission and TLT. In other words,
the benefits of using a multi-criteria optimization approach are in the fact that the method enables us to choose among several near-optimal solutions. The digraph corresponding to solution $x_D$ in Table III is depicted in Fig. 9, the other solution digraphs may easily be obtained from the last column in Table III.

Comparing the results obtained solving the ILP problem with the fuzzy optimization results in [14], we remark that the presented optimization method provides a set of near-optimal solutions instead of only one suboptimal solution. Hence, the designer can be guided by priorities and preferences to choose a satisfactory IESC network, improving system flexibility and agility. Indeed, solution $x_B$ of Table III is the solution obtained in [14] by fuzzy optimization.

### TABLE III.

| Solutions | Costs (US$) | Energy (MJ) | CO$_2$ (KgCE) | TLT (hours) | Indices of variables $x_i=1$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_A$</td>
<td>946.02</td>
<td>6566.30</td>
<td>16.34</td>
<td><strong>98.20</strong></td>
<td>2,4,5,10,14,21,23</td>
</tr>
<tr>
<td>$x_B$</td>
<td>957.02</td>
<td>6269.30</td>
<td>16.36</td>
<td><strong>98.20</strong></td>
<td>2,3,5,10,14,21,23</td>
</tr>
<tr>
<td>$x_C$</td>
<td>964.02</td>
<td>6430.90</td>
<td>14.35</td>
<td>94.80</td>
<td>2,4,6,10,14,18,22</td>
</tr>
<tr>
<td>$x_D$</td>
<td>975.02</td>
<td>6133.90</td>
<td>14.37</td>
<td>94.50</td>
<td>2,3,6,10,14,18,22</td>
</tr>
<tr>
<td>$x_E$</td>
<td>981.60</td>
<td>6437.10</td>
<td>13.24</td>
<td>94.80</td>
<td>2,4,6,10,14</td>
</tr>
<tr>
<td>$x_F$</td>
<td>992.60</td>
<td>6140.10</td>
<td>13.26</td>
<td>94.50</td>
<td>2,3,6,10,14</td>
</tr>
<tr>
<td>$x_G$</td>
<td>1030.92</td>
<td><strong>6112.66</strong></td>
<td>12.51</td>
<td>190.00</td>
<td>2,3,6,10,15,17,18,22</td>
</tr>
<tr>
<td>$x_H$</td>
<td>1037.50</td>
<td>6415.86</td>
<td><strong>11.38</strong></td>
<td>190.30</td>
<td>2,4,6,10,15,17</td>
</tr>
<tr>
<td>$x_I$</td>
<td>1048.50</td>
<td>6118.86</td>
<td>11.40</td>
<td>190.00</td>
<td>2,3,6,10,15,17</td>
</tr>
</tbody>
</table>

Fig. 11. Digraph representing solution $x_D$ of Problem 2 of case study 1.

### V. CASE STUDY 2

To improve the flexibility and agility of the SC, the structure of the IESC of case study 1 is modified by introducing in the second stage three manufacturers and in the fifth stage three consumers. Indeed, the possibility of choosing from one to several manufacturers allows us to obtain a more efficient and reliable IESC. In addition, considering a number of consumers ranging from one to three appears a more realistic situation than the one modeled by case study 1. The digraph describing the new system is
depicted in Fig. 12 and links with the data sheet of the performance indices associated with the digraph edges are reported in Table A1.I of Appendix 1. As explained in Section III, subsection B, the IESC model of this case study includes six fictitious nodes, i.e., \( n_{f_5}, n_{f_6} \) and \( n_{f_7} \), associated with manufacturers \( n_5, n_6 \) and \( n_7 \) respectively, and \( n_{f_{12}}, n_{f_{13}} \) and \( n_{f_{14}} \), associated with consumers \( n_{12}, n_{13} \) and \( n_{14} \), respectively. Hence, the fictitious edges with labels \( x_{64}, x_{65} \) and \( x_{66} \), i.e., arcs connecting \( n_{f_5} \) to \( n_5 \), \( n_{f_6} \) to \( n_6 \) and \( n_{f_7} \) to \( n_7 \), respectively, are considered. Analogously, edges corresponding to variables \( x_{67}, x_{68} \) and \( x_{69} \) are the fictitious arcs connecting \( n_{f_{12}} \) to \( n_{12} \), \( n_{f_{13}} \) to \( n_{13} \) and \( n_{f_{14}} \) to \( n_{14} \), respectively. Moreover, the constraints for Problem 1 and Problem 2 are defined considering two different situations: only one manufacturer and only one consumer are selected (case study 2A) and two manufacturers and two consumers are selected (case study 2B).

Fig. 12. The digraph associated with the IESC of case study 2.

The multi-criteria linear program is the following:

\[ z = \min f(x) = Cx \quad (21) \]

subject to

\[ Ax \geq B \quad (22) \]

\[ x_h \in \{0,1\} \text{ for } h=1,\ldots,E \quad (23) \]

with \( E=69 \), but no performance index is assigned to each fictitious variable \( x_h \) for \( h=64,\ldots,69 \).
TABLE IV.
THE VALUES OF THE PERFORMANCE INDICES FOR PROBLEM 2 OF CASE STUDY 2A:
OPTIMIZATION RESULTS FOR MULTI-OBJECTIVE FUNCTION COSTS, ENERGY AND CO\textsubscript{2} EMISSION.

<table>
<thead>
<tr>
<th>Optimal solutions</th>
<th>Costs (US$)</th>
<th>Energy (MJ)</th>
<th>CO\textsubscript{2} emission (KgCE)</th>
<th>Indices of variables $x_2=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_A$</td>
<td>930.42</td>
<td>7204.80</td>
<td>348.77</td>
<td>25,27,32,45,54,63</td>
</tr>
<tr>
<td>$X_B$</td>
<td>932.12</td>
<td>6371.80</td>
<td>14.21</td>
<td>2,25,27,30,32,51,53</td>
</tr>
<tr>
<td>$X_C$</td>
<td>951.70</td>
<td>6377.00</td>
<td>13.14</td>
<td>2,25,27,30,51</td>
</tr>
<tr>
<td>$X_D$</td>
<td>967.12</td>
<td>6156.80</td>
<td>14.31</td>
<td>2,24,27,30,32,51,53</td>
</tr>
<tr>
<td>$X_E$</td>
<td>972.12</td>
<td>6133.80</td>
<td>14.39</td>
<td>2,3,6,10,22,51,53</td>
</tr>
<tr>
<td>$X_F$</td>
<td>1009.40</td>
<td>6356.18</td>
<td>11.24</td>
<td>2,15,25,27,30,50</td>
</tr>
<tr>
<td>$X_G$</td>
<td>1029.82</td>
<td>6112.98</td>
<td>12.49</td>
<td>2,3,6,10,15,22,50,54</td>
</tr>
<tr>
<td>$X_H$</td>
<td>1030.92</td>
<td>\textbf{6112.66}</td>
<td>12.51</td>
<td>2,3,6,10,15,17,18,22</td>
</tr>
<tr>
<td>$X_I$</td>
<td>1047.40</td>
<td>6119.18</td>
<td>\textbf{11.38}</td>
<td>2,3,6,10,15,50</td>
</tr>
<tr>
<td>$X_J$</td>
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<td>11.40</td>
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The constraints (22) for case study 2A and 2B are listed in Appendix 2. Moreover, Table IV and Table V report the values of the performance indices obtained minimizing costs, energy and CO\textsubscript{2} emission for case study 2A and 2B, respectively. An analysis of Table IV shows the benefits of the multi-criteria optimization approach, which enables us to choose among several near-optimal solutions. In particular, we obtain solution $X_F$ in Table IV performing a good value of costs, energy and CO\textsubscript{2} emission for case study 2A, and the corresponding network is depicted in Fig. 13. In addition, Table V shows that solution $X_F$ performs satisfactorily in terms of costs, energy and CO\textsubscript{2} emission: the corresponding digraph is depicted in Fig. 14. Note that the digraph is composed by two parallel productive chains. In this case a remarkable advantage is observed: if a transporter is temporarily unavailable and/or a communication way cannot be momentarily employed, the productive cycle does not stop. In addition, the considered digraph exhibits edges $y_{15,6}$ and $y_{18,3}$ associated to two recycling
links. It is evident that such links serve the purpose of facilitating component recycling and of extracting hazardous components from products to ensure environmental safety. A consequence of the presence of such de-manufacturing connections is the reduction of the overall costs and CO\textsubscript{2} emission performance indices in the selected IESC configuration.

**TABLE V.**

THE VALUES OF THE PERFORMANCE INDICES FOR PROBLEM 2 OF CASE STUDY 2B: OPTIMIZATION RESULTS FOR MULTI-OBJECTIVE FUNCTION COSTS, ENERGY AND CO\textsubscript{2} EMISSION.

<table>
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<tr>
<th>Optimal solutions</th>
<th>Costs (US$)</th>
<th>Energy (MJ)</th>
<th>CO\textsubscript{2} emission (KgCE)</th>
<th>Indices of variables (x_i=1)</th>
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<td>69.03</td>
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VI. CONCLUSIONS

This paper proposes a network design configuration strategy to select the stage actors of Integrated E-Supply Chains (IESC), that are complex distributed manufacturing systems incorporating the power of e-commerce and electronic information. The network structure of the IESC is modeled by a digraph describing the stage partners and the material and information links connecting them. Moreover, a procedure to define the constraints and the single- and multi-objective optimization problem is stated on the basis of the digraph knowledge. The main peculiarity of the proposed methodology is its flexibility in building the optimization constraints: this characteristic improves agility, environmental performance, and reconfigurability in the IESC network design. Indeed, it is possible to add or substitute constraints that can represent the presence or absence of a transportation mode and of an e-commerce connection. Moreover, the objective function can be easily selected and modified.

The proposed optimization strategy is applied to two case studies inspired by an IESC producing desktop computers described in [14]. The multi-criteria optimization problem solution proposes different structures for the IESC on the basis of the performance indices associated with the material and information links. Moreover, to improve agility in the IESC network design, some networks are obtained selecting from the corresponding sets of candidates only one or two manufacturers and only one or two consumers. The presented research clearly advances state-of-the-art in the area of integrated supply chain design and shows great promise for industrial applications.
### APPENDIX I

#### TABLE A1.I.

Data sheet for the network links in Case Study 2.

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<tr>
<th>Links</th>
<th>Edges</th>
<th>Variables</th>
<th>Costs (M₃) in US$</th>
<th>Energy (M₂) in MJ</th>
<th>CO₂ emission (M₄) in KgCE</th>
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### APPENDIX 2

#### A. Constraints of Case Study 2A.

The constraints for case study 2A may be obtained in a similar way as for case study 1 (see Section III).

**BOM constraints.** The BOM constraints of the second stage are defined as for case study 1: C is produced by \(n_1\) and \(n_2\), H is produced by \(n_1\), \(n_2\) and \(n_3\), M is produced by \(n_2\), \(n_3\) and \(n_4\), and K is produced by \(n_3\) and \(n_4\). Then, the obtained constraints imposed on the edges are the following:

\[
\begin{align*}
  &x_3 + x_4 - x_{64} \geq 0 \\
  &x_3 + x_4 + x_5 - x_{64} \geq 0 \\
  &x_4 + x_5 + x_6 - x_{64} \geq 0 \\
  &x_5 + x_6 - x_{64} \geq 0 \\
  &x_{24} + x_{25} - x_{65} \geq 0 \\
  &x_{24} + x_{25} + x_{26} - x_{65} \geq 0 \\
  &x_{25} + x_{26} + x_{27} - x_{65} \geq 0 \\
  &x_{26} + x_{27} - x_{65} \geq 0 \\
  &x_{33} + x_{34} - x_{66} \geq 0 \\
  &x_{33} + x_{34} + x_{35} - x_{66} \geq 0 \\
  &x_{34} + x_{35} + x_{36} - x_{66} \geq 0 \\
  &x_{35} + x_{36} - x_{66} \geq 0
\end{align*}
\]

(24)

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<td>(m_{11,16})</td>
<td>(y_{11,16})</td>
<td>(x_{54})</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
</tr>
<tr>
<td>(m_{11,17})</td>
<td>(y_{11,17})</td>
<td>(x_{55})</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
</tr>
<tr>
<td>(m_{11,18})</td>
<td>(y_{11,18})</td>
<td>(x_{56})</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
</tr>
<tr>
<td>(m_{12,15})</td>
<td>(y_{12,15})</td>
<td>(x_{57})</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
</tr>
<tr>
<td>(m_{12,16})</td>
<td>(y_{12,16})</td>
<td>(x_{58})</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
</tr>
<tr>
<td>(m_{12,17})</td>
<td>(y_{12,17})</td>
<td>(x_{59})</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
</tr>
<tr>
<td>(m_{12,18})</td>
<td>(y_{12,18})</td>
<td>(x_{60})</td>
<td>0.42</td>
<td>4.80</td>
<td>0.37</td>
</tr>
<tr>
<td>(m_{13,15})</td>
<td>(y_{13,15})</td>
<td>(x_{61})</td>
<td>-19.00</td>
<td>-12.00</td>
<td>0.80</td>
</tr>
<tr>
<td>(m_{13,16})</td>
<td>(y_{13,16})</td>
<td>(x_{62})</td>
<td>60.00</td>
<td>370.00</td>
<td>32.00</td>
</tr>
<tr>
<td>(m_{14,16})</td>
<td>(y_{14,16})</td>
<td>(x_{63})</td>
<td>50.00</td>
<td>380.00</td>
<td>39.00</td>
</tr>
</tbody>
</table>
Path and structural constraints. It is necessary to select in the digraph a path that starts from a node in the manufacturer stage and ends in a node of the consumer stage. For the sake of simplicity, these constraints are expressed as structural constraints derived from the path requirements:

\[
\begin{align*}
  x_9 + x_{29} + x_{40} + x_{16} + x_{14} + x_{17} - x_{67} &\geq 0 \\
  x_{41} + x_{42} + x_{43} + x_{47} + x_{51} + x_{49} - x_{68} &\geq 0 \\
  x_{44} + x_{45} + x_{46} + x_{48} + x_{52} + x_{50} - x_{69} &\geq 0 \\
  x_8 + x_9 + x_{41} + x_{44} + x_{10} + x_{11} - x_{64} &= 0 \\
  x_{28} + x_{29} + x_{42} + x_{45} + x_{30} + x_{31} - x_{65} &= 0 \\
  x_{12} - x_8 - x_{28} - x_{38} &= 0 \\
  x_{13} + x_{14} + x_{51} + x_{52} + x_{15} - x_{10} - x_{30} - x_{37} &\geq 0 \\
  x_{16} + x_{47} + x_{48} - x_{12} - x_{13} &\geq 0 \\
  x_{17} + x_{49} + x_{50} - x_{15} - x_{11} - x_{31} - x_{39} &\geq 0
\end{align*}
\] (25)

In addition, the structural constraints are as follows:

\[
\begin{align*}
  x_5 + x_{26} + x_{35} - x_{23} &\geq 0 \\
  x_{23} - x_{21} - x_{59} - x_{60} &= 0 \\
  x_{64} - x_{22} &\geq 0 \\
  x_{65} - x_{32} &\geq 0 \\
  x_{66} - x_{61} &\geq 0 \\
  x_{67} - x_{18} &\geq 0 \\
  x_{68} - x_{53} &\geq 0 \\
  x_{69} - x_{54} &\geq 0 \\
  x_7 - x_{67} &\geq 0 \\
  x_{68} - x_{62} &\geq 0 \\
  x_{69} - x_{63} &\geq 0 \\
  x_{48} + x_{16} + x_{47} - x_{1} &\geq 0 \\
  x_{52} + x_{13} + x_{15} + x_{14} + x_{51} - x_{2} &\geq 0 \\
  x_{67} - x_{19} - x_{20} - x_{21} - x_{18} &\geq 0 \\
  x_{68} - x_{55} - x_{57} - x_{59} - x_{53} &\geq 0 \\
  x_{69} - x_{56} - x_{58} - x_{60} - x_{54} &\geq 0
\end{align*}
\] (26)

Mutual exclusion constraints. We impose that the stages of recyclers, consumers and manufacturers are composed by only one actor each. Then the mutual exclusion constraints are the following:
\[ x_1 + x_2 + x_7 + x_{62} + x_{63} = 1 \]
\[ x_{64} + x_{65} + x_{66} = 1 \]
\[ x_{67} + x_{68} + x_{69} = 1 \]  

(27)

**B. Constraints of Case Study 2B.**

*BOM constraints.* The BOM constraints are identical to the BOM constraints expressed by (24) of case study 2A.

*Path and structural constraints.* It is necessary to select in the digraph a path that starts from a node of the manufacturer stage and ends to a node of the consumer stage. For the sake of simplicity, these constraints are expressed as structural constraints derived from the path requirements:

\[ x_9 + x_{29} + x_{40} + x_{16} + x_{14} + x_{17} - x_{67} \geq 0 \]
\[ x_{41} + x_{42} + x_{43} + x_{47} + x_{51} + x_{49} - x_{68} \geq 0 \]
\[ x_{44} + x_{45} + x_{46} + x_{48} + x_{52} + x_{50} - x_{69} \geq 0 \]
\[ x_8 + x_9 + x_{41} + x_{44} + x_{10} + x_{11} - x_{64} = 0 \]
\[ x_{28} + x_{29} + x_{42} + x_{45} + x_{30} + x_{31} - x_{65} = 0 \]
\[ x_{38} + x_{40} + x_{43} + x_{46} + x_{37} + x_{39} - x_{66} = 0 \]
\[ x_{12} - x_8 - x_{28} - x_{38} = 0 \]  

(28)

\[ x_{13} + x_{14} + x_{51} + x_{52} + x_{15} - x_{10} - x_{30} - x_{37} \geq 0 \]
\[ x_{16} + x_{47} + x_{48} - x_{12} - x_{13} \geq 0 \]
\[ x_{17} + x_{49} + x_{50} - x_{15} - x_{11} - x_{31} - x_{39} \geq 0 \]
\[ x_{16} + x_{47} + x_{48} - x_{13} \geq 0 \]
\[ x_{17} + x_{49} + x_{50} - x_{15} \geq 0 \]

In addition, the structural constraints are the following:
\[ x_{35} + x_5 + x_{26} - x_{23} \geq 0 \\
x_{23} - x_{21} - x_{59} - x_{60} = 0 \\
x_{64} - x_{22} \geq 0 \\
x_{65} - x_{32} \geq 0 \\
x_{66} - x_{61} \geq 0 \\
x_{67} - x_{18} \geq 0 \\
x_{68} - x_{53} \geq 0 \\
x_{69} - x_{54} \leq 0 \\
x_{22} + x_{32} + x_{61} - x_{18} - x_{53} - x_{54} = 0 \\
x_7 + x_{14} + x_{16} + x_{17} - x_{67} \geq 0 \\
x_{62} + x_{47} + x_{51} + x_{49} - x_{68} \geq 0 \\
x_{63} + x_{48} + x_{52} + x_{50} - x_{69} \geq 0 \\
x_{67} - x_{19} - x_{20} - x_{21} - x_{18} \geq 0 \\
x_{68} - x_{55} - x_{57} - x_{59} - x_{53} \geq 0 \\
x_{69} - x_{56} - x_{58} - x_{60} - x_{69} \geq 0 \\
x_{48} + x_{16} + x_{47} - x_1 \geq 0 \\
x_{52} + x_{13} + x_{15} + x_{14} + x_{51} - x_2 \geq 0 \] (29)

**Mutual exclusion constraints.** We impose that the recycler stage is composed by at most two actors:

\[ x_1 + x_2 + x_7 + x_{62} + x_{63} = 2 \] (30)

Finally, to choose two manufacturers and two consumers the following constraints are respectively imposed:

\[ x_{64} + x_{65} + x_{66} = 2 \]

\[ x_{67} + x_{68} + x_{69} = 2 \] (31)

**REFERENCES**


