Sliding-Mode Control With Double Boundary Layer for Robust Compensation of Payload Mass and Friction in Linear Motors

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Abstract—Direct drives with linear motors have been recently attracting the attention of both industry and academia. The main peculiarity of these systems is the lack of mechanical reduction and transmission devices, which makes the influence of some uncertain electromechanical phenomena (e.g., friction, cogging forces, etc.) and load disturbances much more significant than in the case of conventional rotary actuators. This paper describes a control system for a tubular synchronous linear motor based on a sliding-mode control (SMC) and a proportional–integral (PI)-based equivalent disturbance observer. The distinctive peculiarities of the proposed scheme are the use of a control law that guarantees the stability of the system regardless of the payload mass, the adoption of a double boundary layer addressing effectively the harmful effects of static friction, and the introduction of a simple PI-based equivalent disturbance observer that avoids steady-state errors regardless of model uncertainties and external disturbances. The reduced computational cost of the control law, alongside with the introduction of the effective design criteria for the SMC and the disturbance observer, makes the implementation of the proposed approach as simple as standard cascaded linear control schemes using industrial microcontrollers. The aforementioned considerations are validated by extensive experiments.

Index Terms—Robust control, sliding-mode control (SMC), tubular linear synchronous motor (TLSM), vector control.

I. INTRODUCTION

DIRECT-DRIVE linear motors are becoming increasingly widespread in miniaturized robotics and automation applications. The main peculiarity of these systems is the lack of mechanical reduction and transmission devices. On the one hand, the absence of equipment such as gears and lead screws eliminates typical nonlinearities such as backlash or structural deformations. This permits obtaining higher precision, higher acceleration/deceleration, and reduced dimensions with respect to rotary actuators. On the other hand, the lack of reduction gears makes the influence of load disturbances and various uncertain electromechanical phenomena (e.g., friction, cogging forces, etc.) much more significant than in the case of conventional rotary actuators. Moreover, direct drive linear motors are affected by mechanical resonances, particularly at high-acceleration/deceleration regimes, which vary with different operating conditions and from machine to machine [1].

Due to these reasons, in the last decade, a considerable amount of research work has been devoted to effectively control this type of actuators. The simplest scheme, yet extremely common in industrial practice, is based on one or multiple cascaded proportional–integral–derivative control loops [2], [3]. Notoriously, these schemes are quite common due to the relative ease of configuration, but they are often unable to fulfill the strict specifications of a precision control in micrometric positioning systems. In fact, due to friction, the integral action may generate limit cycles around the target position (the hunting effect) [3], [4], while a proportional–derivative law is affected by steady-state errors. Increasing the controller gains may be beneficial to mitigate or even eliminate these problems, but it will also reduce the stability margins. One way of improving system performance is to augment the control action with a friction compensation term based on a parametric model of the friction phenomena. However, it is generally difficult to determine a priori a friction model with the accuracy required for effective cancellation [5], and, often, friction characteristics vary with time, temperature, and other uncertain factors that cannot explicitly be accounted for during the controller design. Similar difficulties arise in the case of load disturbances.

In recent years, sliding-mode control (SMC) has met a growing interest in many industrial applications, including electrical drives [6], [7]. The attention toward this technique is due to its inherent robustness to parameter uncertainty, insensitivity to load disturbance, and fast dynamics response. However, it is also well known that these properties hold true on the sliding surface only, assuming an ideal control action. On the other hand, the direct implementation of standard SMC laws may exhibit high-frequency oscillations of both controller output and state variables known as chattering. In electrical drives, this phenomenon may cause various undesirable effects such as current harmonics, torque pulsation, and acoustic noise. Many ways to cope with chattering have been proposed. A control engineer’s guide to SMC, which includes a discussion about chattering elimination, is available in [8]. A common method consists in the substitution of the switching element...
Fig. 1. Block diagram of the drive.

by a saturation function so that a transition or boundary layer is introduced around the sliding surface [7], [9]. Although various advanced chattering elimination strategies have been proposed in the recent literature (see [10]), the boundary layer method remains an interesting and straightforward design tool, which often allows users to easily obtain the desired tradeoff between control performances (tracking and robustness) and the reduction of chattering.

This paper proposes an SMC scheme for a tubular linear synchronous motor (TLSM) aiming at a highly accurate and robust position tracking with micrometric tolerances. To achieve robustness to parametric uncertainties and load disturbances, differently from conventional sliding-mode schemes, the proposed control system varies its action by scheduling reaching and sliding actions according to various conditions related to the state trajectory in the phase plane. In particular, a double boundary layer is designed to simultaneously cope with chattering and static friction, both constituting key issues for any realistic implementation of micrometric positioning systems. Finally, a proportional–integral (PI)-based equivalent disturbance observer is introduced to guarantee zero steady-state errors for constant references and external disturbances.

The remainder of this paper is organized as follows. Section II introduces the model of a TLSM and the related assumptions. Section III describes the SMC scheme in various sections, each devoted to one component of the control law. Section IV discusses the main design guidelines for the proposed SMC and summarizes the experimental results obtained on a commercially available TLSM. The final remarks are drawn in Section V.

II. LINEAR SYNCHRONOUS MOTORS: MODEL AND ASSUMPTIONS

The TLSM adopted in this study is a three-phase linear motor, which includes a mover containing the three-phase windings and a tubular rod containing the permanent magnets. The permanent magnets are cylindrically shaped, axially magnetized, and uniformly distributed to form an alternate sequence of magnets and spacers. The three-phase windings are wrapped around the rod, and the mover does not contain a magnetic material. This permits exploiting the magnetic flux with good efficiency and avoiding cogging forces. This type of motor is often adopted for high-precision applications, as it can guarantee position resolutions of orders of micrometers and below. The motor is driven by a current-controlled pulsewidth-modulation voltage source inverter.

In the $d$–$q$ reference frame, synchronously moving at $\dot{x}$ m/s, the mover equation of a TLSM is as follows:

$$\tau_{dq} = L\frac{d\tau_{dq}}{dt} + R\tau_{dq} + j\dot{x}\frac{\pi}{\tau_p}\tau_{dq} + j\dot{x}\frac{\pi}{\tau_p}\lambda_f$$  \hspace{1cm} (1)

where $\pi$ is the mover voltage vector, $\tau$ is the mover current vector, $R$ is the mover resistance, $L$ is the mover inductance, $\dot{x}$ is the speed of the mover, $\lambda_f$ is the mover magnet flux linkage, and $\tau_p$ is the pole pitch (corresponding to $\pi$ electrical degrees). To enhance $d$- and $q$-axis current control, the coupling terms in (1) and the back electromotive force have to be compensated through the feedforward injection of the voltage vector

$$\tau_{dq,\text{comp}} = j\dot{x}\frac{\pi}{\tau_p}L\tau_{dq} + j\dot{x}\frac{\pi}{\tau_p}\lambda_f.$$  \hspace{1cm} (2)

The $d$- and $q$-axis current components are then indirectly controlled by the following voltage vector:

$$\tau_{dq} = \tau_{dq} - \tau_{dq,\text{comp}} = L\frac{d\tau_{dq}}{dt} + R\tau_{dq}.$$  \hspace{1cm} (3)

The mathematical model of the TLSM is completed by the mechanical equation

$$M\ddot{x} = F_e - F = \left(\frac{3}{2}\frac{\pi}{\tau_p}\lambda_f i_q\right) - F$$  \hspace{1cm} (4)

where $M$ is the mover and load mass, $F_e$ is the electromagnetic force, and $F$ is the unknown force caused by friction, load forces, and other uncertain phenomena. Fig. 1 shows the block diagram of the drive. Since the maximum force/current control is achieved when the $d$-axis current is zero, we set $d$-axis current reference $i_d^* = 0$. The mover position is measured with...
a linear encoder. Our design scheme is based on the following assumptions.

**Assumption 1:** The external force $F$ is bounded above by the motor rated force, i.e.,

$$|F| \leq K_f i_q^{\text{rat}}$$

where $i_q^{\text{rat}}$ is the rated motor current. In case this assumption is not met, the motor is clearly underdimensioned with respect to the specific application.

**Assumption 2:** The overall mass $M$ of moving equipment (mover and payload) is unknown due to the lack of information about the payload mass, but an upper bound of this quantity is known. More specifically, we assume that the overall mass can be $m$ times the no-payload mass (indicated with $\hat{M}$), i.e.,

$$M \leq m\hat{M}$$

where $m \geq 1$ is a known constant.

**Assumption 3:** The maximum allowable $q$-axis current, defined as $i_q^{\text{max}}$, is $n$ times the rated current ($i_q^{\text{max}} = ni_q^{\text{rat}}$), where $n > 1$ is a known constant.

### III. Proposed Control Scheme

According to standard practice in a TLSM, the $i_d$ and $i_q$ control loops are controlled by two identical PI controllers that make the current transients negligible with respect to the mechanical dynamics. Therefore, it will be assumed hereafter that the references are equal to the actual values during speed and position transients (i.e., $i_d^* = 0$ and $i_q^* = i_q$). Thus, the current $i_q$ will be regarded as the actual control signal.

The proposed approach is focused on the robust control of the mechanical dynamics using a combination of the design tools aiming at different objectives. In particular, the main control action is obtained through an SMC designed to guarantee the asymptotic stability of the closed loop in the presence of unknown uncertainties, such as frictions, payload mass, and measurement noise. To avoid some typical drawbacks of the pure sliding-mode action while preserving a satisfactory robustness to uncertainties, the control law is varied according to a number of conditions related to the system trajectory in the phase plane. In particular, the control action is scheduled using two different boundary layers. Moreover, a disturbance estimator provides a further control term, which progressively augments the SMC action to eliminate steady-state position errors for constant references. The various components of the control sections will be introduced in the next sections.

#### A. Sliding-Mode Position Control

The dynamics in (4) can be rearranged as

$$\ddot{x}(t) = \frac{K_f}{M} i_q(t) - \frac{F}{M}$$

where $K_f = (3/2)(\pi/\tau_p)\lambda_f$ is the force constant.

The design of the SMC law consists of the following two phases:

1) defining an equilibrium surface (the sliding manifold) and a control action such that any state trajectory starting from the equilibrium surface evolves with a predefined behavior;

2) designing a discontinuous control law to force the state trajectory to reach the sliding surface in a finite time.

Let us define the following sliding surface:

$$s_x(x,t) = \dot{x} + \lambda_x e_x = 0$$

where $e_x = x^* - x$, where $x^*$ is the position reference, and $\lambda_x > 0$ is a design parameter defining the error dynamics once the system has reached the sliding surface ($1/\lambda_x$ is the time constant with which the error decays to zero).

Using standard sliding-mode design arguments, it can easily be shown [9] that the phase plane trajectories reach the sliding surface in a finite time if the control is such that the following condition holds true:

$$\dot{s}_x \text{sign}(s_x) \leq -\eta_x$$

where $\eta_x$ is a strictly positive design constant. In the following, we will discuss how the control action $i_q$ can be designed in order to achieve the aforementioned condition.

Inserting (7) and (8) into (9), we obtain

$$\left[\ddot{x}^* - \frac{K_f}{M} i_q + \frac{F}{M} + \lambda_x e_x \right] \text{sign}(s_x) \leq -\eta_x.$$  (10)

Let us introduce the normalized control gain $\alpha = (K_f/M)$. Even assuming precise knowledge of the force constant $K_f$, the normalized gain remains an uncertain quantity because of the lack of knowledge about $M$. Hereafter, we denote the normalized no-payload control gain with $\hat{\alpha}$ (i.e., $\hat{\alpha} = K_f/M$) and define the normalized gain error $\hat{\alpha} = \alpha - \hat{\alpha} \leq 0$. According to these definitions, (10) can be rewritten as

$$\left[\ddot{x}^* - \hat{\alpha} i_q + \frac{F}{M} + \lambda_x e_x \right] \text{sign}(s_x) \leq -\eta_x.$$  (11)

As mentioned, the SMC law is scheduled according to the conditions related to the system trajectory in the phase plane. More specifically, the action is modified according to the distance from the boundary layers of the sliding manifold. Thus, the illustration of the control law follows two consecutive steps: First, we introduce the control law applied when the state trajectory is outside the boundary layer. Once the design condition that makes the boundary layer attractive is established, we focus on the control policy applied when the state trajectory is inside the boundary layer.

#### B. Control Outside the Boundary Layer

The case of the $s_x$ trajectory being outside the boundary layer occurs when the position and/or velocity tracking errors are relatively large. Under these circumstances, since the state trajectory is distant from the reference trajectory, the position set point is hold constant (i.e., $\ddot{x}^* = \dot{x}^* = 0$) until the trajectory enters the boundary layer. The control action applied in this case is

$$i_q = \begin{cases} i_q^{\text{max}} \text{clip}(s_x) - \frac{\lambda_x}{\alpha} \dot{s}_x, & \text{if } \text{sign}(s_x) = \text{sign}(\dot{x}) \\ i_q^{\text{max}} \text{clip}(s_x), & \text{otherwise.} \end{cases}$$  (12)

$$\dot{s}_x$$
According to (12b), when the signs of \( s_x \) and \( \dot{x} \) are discordant, the controller applies the maximum current. In this case, assuming \( s_x > 0 \) implies that \( \dot{x} < 0 \) and \( i_q = i_q^{\text{rat}} \). Thus, according to (7) and to the assumptions listed in Section II, the mover dynamics become

\[
\dot{x}(t) = \frac{K_f}{M} q_{\text{max}} - \frac{F}{M} \geq \alpha_i^{\text{max}} - \frac{|F|}{M} \\
\geq \alpha_i^{\text{max}} - \frac{K_f}{M} i_q^{\text{rat}} = \alpha_i^{\text{max}} - \alpha_i^{\text{rat}} \\
= \alpha(n-1)i_q^{\text{rat}}.
\] (13)

Thus, \( n > 1 \) is a necessary condition to obtain a positive acceleration, causing a speed reversal after a finite time. It can be remarked that, since the TLSM is generally used in applications requiring very fast dynamics, the ratio \( n \) is usually well above unity. Analogous considerations can be formulated for the case \( s_x < 0 \) and \( \dot{x} > 0 \). In both circumstances, the controller forces \( \dot{x} \) to have the same sign of \( s_x \) and then reach the condition described by (12a), under which the amplitude of the applied current is progressively reduced. Integrating (13), we obtain

\[
T_{\text{inv}} = \dot{x}(0)/(\alpha(n-1)i_q^{\text{rat}})
\] (14)

where \( T_{\text{inv}} \) is an upper bound of the time needed to obtain the speed reversal, after which the state is driven toward the sliding surface with a control action that is below its maximum limit. Once the condition \( \text{sign}(s_x) = \text{sign}(\dot{x}) \) is achieved, substituting \( i_q \) from (12a) into (11), we obtain

\[
[-i_q^{\text{max}} \text{sign}(s_x) \dot{\alpha} + d_x^{\text{eq}}] \text{sign}(s_x) \leq -\eta_x
\] (15)

where the equivalent disturbance \( d_x^{\text{eq}} \) is defined as follows:

\[
d_x^{\text{eq}} = -\dot{\alpha}i_q + \frac{F}{M}.
\] (16)

Let \( |d_x^{\text{eq}}| \) have an upper bound with known value \( D_x^{\text{eq}} \). The condition (15) is satisfied if

\[
i_q^{\text{max}} \geq \left( \frac{D_x^{\text{eq}} + \eta_x}{\dot{\alpha}} \right)
\] (17)

i.e., under this condition, the phase plane trajectories head toward the sliding surface and reach the boundary layer in a finite time in spite of the disturbance given by (16). Using the assumptions in Section II, the stability condition (17) can easily be rewritten as follows:

\[
i_q^{\text{max}} \geq \left( \frac{\dot{\alpha} m i_q^{\text{rat}} + m - 1}{m} \dot{\alpha} i_q^{\text{max}} + \eta_x \right)/\dot{\alpha}.
\] (18)

This inequality is satisfied when

\[
\frac{n - 1}{m} i_q^{\text{rat}} \geq \frac{\eta_x}{\dot{\alpha}}.
\] (19)

The last condition shows that having \( n > 1 \) is also a necessary condition to guarantee that the boundary layer is attractive for the state trajectories regardless of the payload mass. Moreover, using standard sliding-mode design arguments, we argue that, outside the boundary layer, the mover goes toward the desired position with a speed whose steady-state value is larger than \( (n-1)(\dot{\alpha}/\lambda_x)i_q^{\text{rat}} \). This can be proven in the following way. Combining (15) and (16), we obtain

\[
\dot{s}_x = -i_q^{\text{max}} \text{sign}(s_x) \dot{\alpha} - \dot{\alpha} i_q + \frac{F}{M}.
\] (20)

Moreover, from the definition of the sliding surface and assuming constant reference

\[
\dot{s}_x = \dot{e}_z + \lambda_x e_z = \ddot{x} - \dot{x} + \lambda_x (\dot{x} - \dot{x}) = -\dot{x} - \lambda_x \dot{x}.
\] (21)

The differential equation describing the speed dynamics is the following:

\[
\ddot{x} + \lambda_x \dot{x} = i_q^{\text{max}} \dot{\alpha} + \dot{\alpha} \left( \frac{i_q^{\text{max}} - \lambda_x \ddot{x}}{\dot{\alpha}} \right) - \frac{F}{M}
\] (22)

Without losing generality, assuming \( s_x < 0 \) and using (12a)

\[
\ddot{x} + \dot{x} \frac{\dot{\alpha}}{\lambda_x} = i_q^{\text{max}} \dot{\alpha} - \frac{F}{M}
\] (24)

Therefore, the speed tends to the steady-state value given by

\[
\dot{x}^* = \left( \frac{i_q^{\text{max}} - \frac{F}{M} \dot{\alpha}}{\lambda_x} \right) > \left( \frac{i_q^{\text{rat}} \dot{\alpha} - \frac{K_f}{M \dot{\alpha}} \dot{\alpha}}{\lambda_x} \right) \frac{1}{\lambda_x}
\] (25)

with a time constant that is equal to \( \dot{\alpha}/(\alpha \lambda_x) \), which is lower than \( m/\lambda_x \). The value assigned to parameter \( \lambda_x \) determines the minimum speed during the reaching phase (outside the boundary layer). The reaching time will be lower than the sum of \( 3m/\lambda_x \) (maximum time needed to reach 95% \( \dot{x}^* \) speed) and the ratio between the maximum position error (the possible movement of the motor) and \( \dot{x}^* \). A similar analysis can be repeated in the case of negative \( s_x \). The steady-state speed has always the same sign of \( s_x \), thus justifying the use of (12a) in the previous mathematical analyses. During the reaching period (when the speed is constant), the control action must only compensate for load force and friction, and, therefore, the current should be well within its limits. This design choice also ensures smooth transients in case of large position errors, as confirmed by the experimental investigations discussed in the next section.

C. Control Inside the Boundary Layer

Under the control action designed in the previous section, the motor does not reach the ideal sliding-mode dynamics \( \dot{s}_x = 0 \). In fact, comparing (9) and (15), we obtain

\[
\dot{s}_x = -i_q^{\text{max}} \text{sign}(s_x) \dot{\alpha} + d_x^{\text{eq}}
\] (26)

as long as the controls (12a) and (12b) are applied. Moreover, the discontinuous control action \( i_q^{\text{rat}} \) across the surface \( s_x = 0 \)
generates chattering. To reduce the effect of this undesirable phenomenon, a modified saturation function \( \text{sat}(s_x, \Phi_h, \Phi_l, \beta) \) is used to replace the switching function \( \text{sign}(s_x) \) in (12) and (26). The saturation function is defined as follows:

\[
\text{sat}(s_x, \Phi_h, \Phi_l, \beta) = \begin{cases} 
\text{sign}(s_x), & |s_x| \leq \Phi_h \\
\frac{2s_x}{\Phi_h} + \beta \cdot \text{sign}(s_x), & \Phi_h < |s_x| \leq \Phi_l \\
\frac{2s_x}{\Phi_l} + \beta \cdot \frac{s_x}{\Phi_l^2}, & \Phi_l < |s_x|
\end{cases}
\]

where \( \Phi_h \) and \( \Phi_l \) are the widths of the outer and the inner boundary layers (\( \Phi_h > \Phi_l > 0 \)), respectively, and \( \beta \) is a design parameter \( 0 < \beta < 1 \). It can be noted that the function \( \text{sat}(s_x, \Phi_h, \Phi_l, \beta) \) is equal to \( \text{sign}(s_x) \) outside the outer boundary layer and is a linear function of \( s_x \) within the inner boundary layer. In particular, the inner layer has a larger gain, while the outer boundary layer includes the further term \( \beta \cdot \text{sign}(s_x) \) to contrast the effects of static friction. Fig. 2 shows the boundary layers and the sliding surface in the state plane.

Introducing the boundary layers, the dynamics while in sliding mode becomes

\[
\dot{s}_x = -i_q^{\max} \cdot \text{sat}(s_x, \Phi_h, \Phi_l, \beta) \cdot \dot{\alpha} + d_{eq}^{\text{eq}}
\]

and the control law for the \( q \)-axis current becomes

\[
i_q = i_q^{\max} \text{sat}(s_x, \Phi_h, \Phi_l, \beta) + \frac{x^*}{\alpha} + \frac{\lambda_x (\dot{x}^* - \dot{x})}{\alpha}
\]

When the state reaches and enters the inner boundary layer, the control law (29) causes the following sliding-mode dynamics [from (26)]:

\[
\dot{s}_x + i_q^{\max} \alpha \left( \frac{1}{\Phi_h} + \frac{\beta}{\Phi_l} \right) s_x = d_{eq}^{\text{eq}}.
\]

This equation emphasizes that the variable \( s_x \), which is a measure of the algebraic distance to the sliding surface, can also be viewed as the output of a first-order filter having the equivalent disturbance as input and the break frequency \( f_b = i_q^{\max} \alpha (1/\Phi_h) + (\beta/\Phi_l) \).  

\[ \text{D. Equivalent Disturbance Estimator} \]

A PI regulator can be used to reduce the filter bandwidth and guarantee zero steady-state error at the same time. Fig. 3 shows the block diagram of the resulting system inside the inner boundary layer. The PI control loop bandwidth can be tuned by suitable selection of \( K_p \) and \( K_i \). The PI output is

\[
\frac{\hat{d}_{eq}^{\text{eq}}}{\alpha} = K_p s_x + K_i \int s_x \, dt
\]

where \( \hat{d}_{eq}^{\text{eq}} \) is the estimate of the equivalent disturbance \( d_{eq}^{\text{eq}} \). The PI contribution is activated only when the state enters the outer boundary layer. In this way, the following can easily be noted: 1) the dynamics outside the outer boundary layer are not changed; 2) the required control action is always lower than \( i_q^{\max} \); and 3) the steady-state error is zero. To sum up, when the state enters the outer boundary layer, the overall control action is

\[
i_q = i_q^{\max} \text{sat}(s_x, \Phi_h, \Phi_l, \beta) + \frac{x^*}{\alpha} + \frac{\lambda_x (\dot{x}^* - \dot{x})}{\alpha} + K_p s_x + K_i \int s_x \, dt.
\]

\[ \text{IV. DESIGN CRITERIA AND EXPERIMENTS} \]

The test bench utilizes a TLSM having the following rated specifications: rated \( i_{sq} \) current is 2.8 A, \( R = 12.03 \, \Omega \), \( L = 7.8 \, \text{mH}, \tau_p = 25.6 \, \text{mm}, K_f = 31.2 \, \text{N/A}, \) mass of the mover is 2.75 kg, and maximum speed is 5.2 m/s (see Fig. 4).

\[ \text{A. Discussion of Design Criteria} \]

In order to implement the proposed control strategy, the first parameter to be selected is \( \lambda_x \), which is the inverse of the time constant of the state dynamics on the sliding surface. Intuitively, the parameter \( \lambda_x \) should be smaller for systems with larger mass values. Choosing the time constant to be equal to 5 ms gives \( \lambda_x = 200 \, \text{s}^{-1} \).

The sizes of the boundary layers \( \Phi_h \) and \( \Phi_l \) should be set by finding a compromise between the need to reduce the filter bandwidth to mitigate chattering and the need to limit
Thus obtaining \( \tau \) system, the PI integral gain is set as choosing the PI time constant in Fig. 3 suggests the possibility of a pole-zero cancellation, for the selection of the PI controller gains. The block diagram ratio \( \Phi \) of these parameters is not necessary.

For the duration of our controller, it was also observed that a fine tuning removed by setting \( \beta \) error is about 50 \( \mu \)m. It can also be noted that the ratio \( \Phi_l/\lambda_x \) is the horizontal distance of the inner boundary layer from the sliding surface (see Fig. 2). By applying this tuning criterion, the inner boundary layer provides an effective action against the effects of static friction.

Outside the inner boundary layer, the control action \( \beta \cdot \text{sign}(s_x) \) should be as close as possible to the ratio between static friction and force constant. In our system, the static friction is around 4.5 N, and \( \beta \) is chosen to be equal to 0.054, thus obtaining \( \tau_0 = 1/f_b = 0.0053 \) s. This value is also useful for the selection of the PI controller gains. The block diagram in Fig. 3 suggests the possibility of a pole-zero cancellation, choosing the PI time constant \( \tau_\text{PI} = K_p/\tau_1 \) to be equal to \( \tau_0 \). The resulting closed-loop system has a real pole with time constant \( \tau_x = 1/(\hat{\alpha} K_p) \). This time constant has to be necessarily larger than \( \tau_0 \) to avoid interferences between the dynamics of the SMC (faster) and the PI (slower). By selecting \( \tau_x = 4\tau_0 \), it is possible to obtain the value of the proportional gain \( K_p = 1/(4\tau_0\hat{\alpha}) \approx 4 \). According to these remarks, in our system, the PI integral gain is set as \( K_i = K_p/\tau_0 \approx 750 \).

Considering these design guidelines, it can easily be remarked that the proposed control system can be obtained with a configuration effort that is comparable to the one needed to design the typical linear cascaded control loops widely adopted in industrial applications of linear drives.

**B. Summary of Experimental Results**

The reference position trajectory is obtained by filtering consecutive steps of variable amplitude with a nonlinear filter that shapes its output to keep the maximum speed and acceleration within the selected limits [11]. In the first set of experiments presented here, the maximum acceleration was chosen to be equal to 4 m/s\(^2\) and the maximum speed to be equal to 0.5 m/s (see Fig. 5).

Fig. 6 compares the position tracking errors obtained with a traditional cascaded control scheme and linear controllers with those given by the proposed SMC scheme. In particular, three tests are presented: without payload, with 6 kg, and with 10 kg of additional payload. The performances of the linear control scheme are rapidly altered by the increased mass, while the tracking error of the SMC scheme is almost not affected by the mass change. Note that a 10-kg payload means that the mass is above 4.6 times the rated value used to tune the control schemes. It is also important to underline that the results obtained with the linear controllers in the no-payload case could be improved by increasing the controller gains, but this option would compromise the performances of the linear scheme with large payloads. The proposed control scheme (using constant parameters) dramatically improves the robustness to mass changes. In order to evidence the efficacy of the double boundary layer and the effect of the parameter \( \beta \), Fig. 7 shows the position tracking errors obtained with a 10-kg payload with a single boundary layer (\( \beta = 0 \)) and with the double boundary layer (\( \beta = 0.054 \)). It is easy to note that not only the latter scheme has a reduced error when the position set point is kept constant but also reduces the position error peaks.

The influence of the trajectory parameters on the performances of the proposed SMC scheme has been investigated by increasing the maximum acceleration up to 10 m/s\(^2\) and the maximum speed up to 1.5 m/s. The sample results shown in Fig. 8 have been obtained with an 8-kg payload and different maximum speeds. The performances of the SMC scheme are only weakly affected by the maximum speed value.

The disturbance rejection is also investigated with an experiment in which a rated load force step is applied during a zero-speed test. The comparison of the position error obtained using the SMC and the linear control is reported in Fig. 9, which provides further evidence of the advantage of the nonlinear control approach.

The main system parameters affecting the speed and position control loops are the force constant, the payload mass, and the current loop bandwidth. The ratio between force constant and payload mass defines the open-loop gain of the plant denoted with \( \alpha \) in the previous sections. We have only considered the variation of the mass value because a mass change can be larger than a force constant change due to temperature effect (force constant decreases with temperature). Moreover, a force constant decrease would produce similar effects to a mass value increase. The current control loop is usually fast enough to neglect its dynamics in the position and speed control loops. Fig. 10 shows the position error obtained using the current control loops tuned so to have different bandwidths. The results clearly show that, within the reasonable large ranges of the current loop bandwidth, the SMC system holds the same overall performance.

Finally, the behavior of the drive outside the boundary layers is investigated using a large square wave (without filtering) as the position reference. Fig. 11 shows that the speed is approximately constant during the transient and equal to 0.18 m/s.
Except for the fast speed transients, the $i_{sq}$ current is well below its limit as predicted by the theoretical remarks drawn in the previous sections.

V. CONCLUSION

This paper has proposed an effective scheme based on sliding mode for the precision control of permanent-magnet linear tubular motors. Even if SMC has been widely adopted in various fields, including the robust control of electrical motors, this paper has introduced some new contributions that improve the performances and facilitate the practical implementation of the nonlinear controller. In particular, this paper has the following original key points.

1) The application of the sliding-mode theory, by means of an original control law, has been adapted at the case study.
of linear synchronous motor control to guarantee stability regardless of the mass of the payload under relatively mild assumptions that are easily met in well-dimensioned applications.

2) The introduction of the double boundary layer reduces the effects of static friction and allows defining general design criteria that simplify the implementation of the proposed control scheme on different motors, including rotating ones.

3) The introduction of a simple PI-based equivalent disturbance observer eliminates steady-state errors.

The low computational cost, together with the effective design criteria suggested in this paper (which are as simple as the classical design rule for linear cascaded control schemes), makes it a competitive alternative for industrial applications with high precision and robustness requirements.

REFERENCES


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